OCR
Oxford Cambridge and RSA

# AS Level Mathematics B (MEI)

## H630/01 Pure Mathematics and Mechanics

# Practice Paper – Set 1

## Time allowed: 1 hour 30 minutes

**You must have:**
- Printed Answer Booklet

**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**
- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

# Formulae AS Level Mathematics B (H630)

## Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

## Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

## The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of $X$ is $np$

## Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Answer **all** the questions.

**1** Solve the equation $\cos 2\theta = 0.3$ for $0° \leqslant \theta < 360°$. **[3]**

**2** James tries to solve the inequality $x^2 - 5x - 14 \geqslant 0$. He writes his answer as

$$\{x : x \leqslant 2\} \cup \{x : x < 7\}.$$

Correct all the errors in his answer. **[3]**

**3** A block of mass $5m$ kg is in equilibrium on a rough horizontal table. It is connected by horizontal light inextensible strings over smooth pulleys to particles of mass $m$ kg and $2m$ kg which hang freely, as shown in Fig. 3.

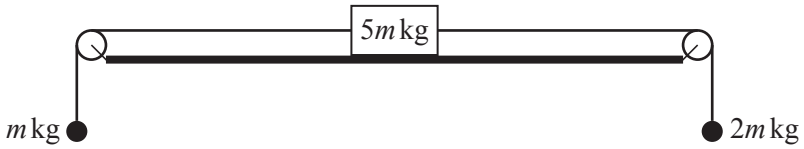


**Fig. 3**

Find the frictional force acting on the block, clearly indicating its direction. **[3]**

**4** A circle has centre C (–4, 3) and passes through the origin.

**(i)** Find the equation of the circle. **[3]**

**(ii)** The circle crosses the positive $y$-axis at the point A. The tangent to the circle at A meets the $x$-axis at B. Find the $x$-coordinate of B. **[5]**

**5** The spreadsheet in Fig. 5 shows a multiplication table. The numbers 35, 36, 35 in the shaded cells are of the form $n^2 - 1$, $n^2$, $n^2 - 1$, where $n = 6$. This pattern can also be seen for the other square numbers on the diagonal of the table in cells B2, C3, …, I9.

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

**Fig. 5**

The spreadsheet can be extended to include larger numbers. Prove that the pattern holds for all integers $n > 1$. **[3]**

**6** **In this question you must show detailed reasoning.**

A boy plays on a path that runs north-south through an origin O. His displacement $x$ metres north of O at time $t$ seconds is given by

$$x = -0.7t^2 + 4t \text{ for } 0 \leqslant t \leqslant 10.$$

**(i)** Determine the direction in which he is moving when $t = 7$. **[3]**

**(ii)** Find the furthest distance from O reached by the boy for $0 \leqslant t \leqslant 10$. **[5]**

**7** OACB is a parallelogram. O is the origin and point A has coordinates (5, 6). Point B has position vector $\mathbf{b} = -2\mathbf{i} - 7\mathbf{j}$.

**(i)** Find the coordinates of point C. **[3]**

M is the midpoint of AB.

**(ii)** Prove that $\overrightarrow{OM} = \overrightarrow{MC}$. **[3]**

**(iii)** Find the exact distance MC. **[2]**

**8** **In this question you must show detailed reasoning.**

Find the total area of the shaded regions shown in Fig. 8, bounded by the line $x = -1$, the $x$-axis and the curve $y = x^3(x-3)$. **[6]**

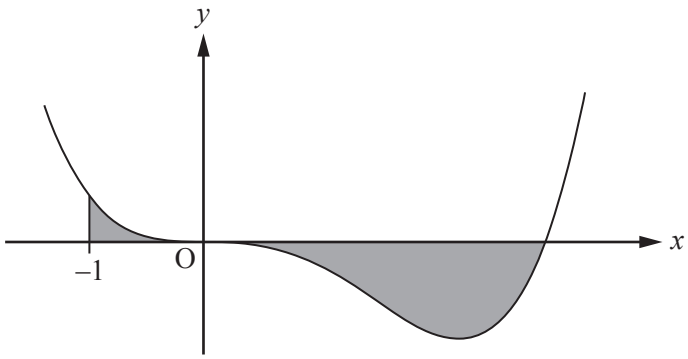


**Fig. 8**

**9** **(i)** Sketch the curve $y = e^{2x}$. **[2]**

**(ii)** Describe fully the transformation that maps the curve $y = e^x$ onto the curve $y = e^{2x}$. **[2]**

**(iii)** Find the equation of the tangent to $y = e^{2x}$ at the point where $x = 3$, giving your answer in the form $y = e^a(bx + c)$ where $a$, $b$ and $c$ are integers. **[6]**

**10** Dorothy and Harold push a wheelbarrow of mass 20 kg. They both push in the direction of motion. The wheelbarrow moves 10 m from rest on a straight horizontal path. Dorothy applies a force of 90 N. Harold applies a force of 90 N for the first 3 m, and then reduces the force to 80 N for the remaining 7 m. The resistance to motion is 170 N throughout.

Find the time taken for the wheelbarrow to move the 10 m. **[9]**

**11** Cheung wishes to model the fuel consumption of a car. He tries the quadratic model

$$y = a(v - b)^2 + c$$

where $y$ is the fuel needed in litres per 100 km and $v$ is the speed in km h$^{-1}$.

Travelling as a passenger, he notices that the minimum fuel consumption displayed on the dashboard is 10 litres per 100 km and occurs at 80 km h$^{-1}$.

**(i)** Write down the values of $b$ and $c$ for which Cheung's model fits this information. **[2]**

At 30 km h$^{-1}$ the fuel consumption displayed is 12 litres per 100 km.

**(ii)** Find the value of $a$ for which Cheung's model fits this information. **[2]**

**(iii)** Use this model to predict the fuel consumption at 110 km h$^{-1}$. **[2]**

**(iv)** Sketch the graph of the model for speeds between 30 km h$^{-1}$ and 110 km h$^{-1}$. **[2]**

Later in the journey, Cheung notices that fuel consumption of 12 litres per 100 km is displayed at 30 km h$^{-1}$ and also at 110 km h$^{-1}$. The minimum fuel consumption still occurs at 80 km h$^{-1}$.

**(v)** Give a reason why a quadratic model cannot fit all the information Cheung has found. **[1]**

**END OF QUESTION PAPER**

**BLANK PAGE**

**BLANK PAGE**