4751 (C1) Introduction to Advanced Mathematics

Section A

1	(i) 0.125 or 1/8 (ii) 1	1 1	as final answer	2
2	$y = 5x - 4$ www	3	M2 for $\frac{y-11}{-9-11} = \frac{x-3}{-1-3}$ o.e. or M1 for grad = $\frac{11-(-9)}{3-(-1)}$ or 5 eg in $y = 5x + k$ and M1 for $y - 11$ = their $m(x - 3)$ o.e. or subst (3, 11) or (−1, −9) in y = their $mx + c$ or M1 for $y = kx - 4$ (eg may be found by drawing)	3
3	$x > 9/6$ o.e. or $9/6 < x$ o.e. www isw	3	M2 for $9 < 6x$ or M1 for $-6x < -9$ or $k < 6x$ or $9 < kx$ or $7 + 2 < 5x + x$ [condone $\le$ for Ms]; if 0, allow SC1 for 9/6 o.e found	3
4	$a = -5$ www	3	M1 for f(2) = 0 used and M1 for $10 + 2a = 0$ or better long division used: M1 for reaching $(8 + a)x - 6$ in working and M1 for $8 + a = 3$ equating coeffts method: M2 for obtaining $x^3 + 2x^2 + 4x + 3$ as other factor	3
5	(i) $4[x^3]$ (ii) $84[x^2]$ www	2 3	ignore any other terms in expansion M1 for $-3[x^3]$ and $7[x^3]$ soi; M1 for $\frac{7\times 6}{2}$ or 21 or for Pascal's triangle seen with 1 7 21 … row and M1 for 2^2 or 4 or $\{2x\}^2$	5

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6	1/5 or 0.2 o.e. www	3	M1 for $3x + 1 = 2x \times 4$ and M1 for $5x = 1$ o.e. <u>or</u> M1 for $1.5 + \frac{1}{2x} = 4$ and M1 for $\frac{1}{2x} = 2.5$ o.e.	3
7	(i) $5^{3.5}$ or $k = 3.5$ or 7/2 o.e. (ii) $16a^6b^{10}$	2 2	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ SC1 for $5^{\frac{3}{2}}$ o.e. as answer without working M1 for two 'terms' correct and multiplied; mark final answer only	4
8	$b^2 - 4ac$ soi $k^2 - 4 \times 2 \times 18 < 0$ o.e. $-12 < k < 12$	M1 M1 A2	allow in quadratic formula or clearly looking for perfect square condone $\le$; or M1 for 12 identified as boundary may be two separate inequalities; A1 for $\le$ used or for one 'end' correct if two separate correct inequalities seen, isw for then wrongly combining them into one statement; condone b instead of k; if no working, SC2 for $k < 12$ and SC2 for $k > -12$ (ie SC2 for each 'end' correct)	4
9	$y + 5 = xy + 2x$ $y - xy = 2x - 5$ oe or ft $y(1 - x) = 2x - 5$ oe or ft $[y =]\frac{2x-5}{1-x}$ oe or ft as final answer	M1 M1 M1 M1	for expansion for collecting terms for taking out y factor; dep on xy term for division and no wrong work after ft earlier errors for equivalent steps if error does not simplify problem	4
10	(i) $9\sqrt{3}$ (ii) $6 + 2\sqrt{2}$ www	2 3	M1 for $5\sqrt{3}$ or $4\sqrt{3}$ seen M1 for attempt to multiply num. and denom. by $3 + \sqrt{2}$ and M1 for denom. 7 or $9 - 2$ soi from denom. mult by $3 + \sqrt{2}$	5

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Section B

11	**i**	C, mid pt of AB = $\left(\frac{11+(-1)}{2}, \frac{4}{2}\right)$ = (5, 2)	B1	evidence of method required – may be on diagram, showing equal steps, or start at A or B and go half the difference towards the other		
		[AB^2 =] $12^2 + 4^2$ [= 160] oe or [CB^2 =] $6^2 + 2^2$ [=40] oe with AC	B1	or square root of these; accept unsimplified		
		quote of $(x - a)^2 + (y - b)^2 = r^2$ o.e with different letters	B1	or (5, 2) clearly identified as centre and $\sqrt{40}$ as r (or 40 as r^2) www or quote of *gfc* formula and finding c = −11		
		completion (ans given)	B1	dependent on centre (or midpt) and radius (or radius^2) found independently and correctly	4	
	ii	correct subst of $x = 0$ in circle eqn soi $(y - 2)^2 = 15$ or $y^2 - 4y - 11$ [= 0] $y - 2 = \pm\sqrt{15}$ or ft [y =]$2 \pm \sqrt{15}$ cao	M1 M1 M1 A1	 condone one error or use of quad formula (condone one error in formula); ft only for 3 term quadratic in y if $y = 0$ subst, allow SC1 for (11, 0) found alt method: M1 for y values are $2 \pm a$ M1 for $a^2 + 5^2 = 40$ soi M1 for $a^2 = 40 - 5^2$ soi A1 for [y =]$2 \pm \sqrt{15}$ cao	4	
	iii	grad AB = $\frac{4}{11-(-1)}$ or 1/3 o.e. so grad tgt = – 3 eqn of tgt is $y - 4 = -3(x - 11)$ $y = -3x + 37$ or $3x + y = 37$ (0, 37) and (37/3, 0) o.e. ft isw	M1 M1 M1 A1 B2	or grad AC (or BC) or ft −1/their gradient of AB or subst (11, 4) in $y = -3x + c$ or ft (no ft for their grad AB used) accept other simplified versions B1 each, ft their tgt for grad ≠ 1 or 1/3; accept $x = 0$, $y = 37$ etc NB alt method: intercepts may be found first by proportion then used to find eqn	6	14

12	**i**	$3x^2 + 6x + 10 = 2 - 4x$ $3x^2 + 10x + 8$ [= 0] $(3x + 4)(x + 2)$ [=0] $x = -2$ or $-4/3$ o.e. $y = 10$ or $22/3$ o.e.	M1 M1 M1 A1 A1	for subst for x or y or subtraction attempted or $3y^2 - 52y + 220$ [=0]; for rearranging to zero (condone one error) or $(3y - 22)(y - 10)$; for sensible attempt at factorising or formula or completing square or A1 for each of $(-2, 10)$ and $(-4/3, 22/3)$ o.e.	5
	ii	$3(x + 1)^2 + 7$	4	1 for $a = 3$, 1 for $b = 1$, 2 for $c = 7$ or M1 for $10 - 3 \times$ their b^2 soi or for 7/3 or for $10/3 -$ their b^2 soi	4
	iii	min at $y = 7$ or ft from (ii) for positive c (ft for (ii) only if in correct form)	B2	may be obtained from (ii) or from good symmetrical graph or identified from table of values showing symmetry condone error in x value in stated min ft from (iii) [getting confused with 3 factor] B1 if say turning pt at $y = 7$ or ft without identifying min <u>or</u> M1 for min at $x = -1$ [e.g. may start again and use calculus to obtain $x = -1$] or min when $(x + 1)^{[2]} = 0$; and A1 for showing y positive at min <u>or</u> M1 for showing discriminant neg. so no real roots and A1 for showing above axis not below eg positive x^2 term or goes though (0, 10) <u>or</u> M1 for stating bracket squared must be positive [or zero] and A1 for saying other term is positive	2

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13	**i**	any correct y value calculated from quadratic seen or implied by plots	B1	for $x \neq 0$ or 1; may be for neg x or eg min.at (2.5, −1.25)		
		(0, 5)(1, 1)(2, −1)(3, −1)(4,1) and (5,5) plotted	P2	tol 1 mm; P1 for 4 correct [including (2.5, −1.25) if plotted]; plots may be implied by curve within 1 mm of correct position		
		good quality smooth parabola within 1mm of their points	C1	allow for correct points only [accept graph on graph paper, not insert]	4	
	ii	$x^2 - 5x + 5 = \frac{1}{x}$	M1			
		$x^3 - 5x^2 + 5x = 1$ and completion to given answer	M1		2	
	iii	divn of $x^3 - 5x^2 + 5x - 1$ by $x - 1$ as far as $x^3 - x^2$ used in working	M1	or inspection eg $(x - 1)(x^2 \ldots.. + 1)$ or equating coeffts with two correct coeffts found		
		$x^2 - 4x + 1$ obtained	A1			
		use of $b^2 - 4ac$ or formula with quadratic factor	M1	or $(x - 2)^2 = 3$; may be implied by correct roots or $\sqrt{12}$ obtained		
		$\sqrt{12}$ obtained and comment re shows other roots (real and) irrational <u>or</u> for $2 \pm \sqrt{3}$ or $\frac{4 \pm \sqrt{12}}{2}$ obtained isw	A2	[A1 for $\sqrt{12}$ and A1 for comment] NB A2 is available only for correct quadratic factor used; if wrong factor used, allow A1 ft for obtaining two irrational roots or for their discriminant and comment re irrational [no ft if their discriminant is negative]	5	11