4751 (C1) Introduction to Advanced Mathematics

Section A

1	(0, 14) and (14/4, 0) o.e. isw	4	M2 for evidence of correct use of gradient with (2, 6) eg sketch with 'stepping' or $y - 6 = -4(x - 2)$ seen or $y = -4x + 14$ o.e. or M1 for $y = -4x + c$ [accept any letter or number] and M1 for $6 = -4 \times 2 + c$; A1 for (0, 14) [$c = 14$ is not sufficient for A1] and A1 for (14/4, 0) o.e.; allow when $x = 0$, $y = 14$ etc isw	4
2	$[a =]\frac{2(s-ut)}{t^2}$ o.e. as final answer [condone $[a =]\frac{(s-ut)}{0.5t^2}$]	3	M1 for each of 3 complete correct steps, ft from previous error if equivalent difficulty [eg dividing by t does not count as step – needs to be by t^2] $[a =]\frac{(s-ut)}{\frac{1}{2}t^2}$ gets M2 only (similarly other triple-deckers)	3
3	10 www	3	M1 for f(3) = 1 soi and A1 for $31 - 3k = 1$ or $27 - 3k = -3$ o.e. [a correct 3-term or 2-term equation] long division used: M1 for reaching $(9 - k)x + 4$ in working and A1 for $4 + 3(9 - k) = 1$ o.e. equating coeffts method: M2 for $(x - 3)(x^2 + 3x - 1)$ [+ 1] o.e. (from inspection or division)	3
4	$x < 0$ or $x > 6$ (both required)	2	B1 each; if B0 then M1 for 0 and 6 identified;	2
5	(i) 10 www (ii) 80 www or ft 8 × their (i)	2 2	M1 for $\frac{5\times4\times3}{3\times2(\times1)}$ or $\frac{5\times4}{2(\times1)}$ or for 1 5 10 10 5 1 seen B2 for $80x^3$; M1 for 2^3 or $(2x)^3$ seen	4
				16

6	any general attempt at n being odd and n being even even n odd implies n^3 odd and odd − odd = even n even implies n^3 even and even − even = even	M1 A1 A1	M0 for just trying numbers, even if some odd, some even or $n(n^2 - 1)$ used with n odd implies $n^2 - 1$ even and odd × even = even etc [allow even × odd = even] or A2 for $n(n - 1)(n + 1)$ = product of 3 consecutive integers; at least one even so product even; odd^3 − odd = odd etc is not sufft for A1 SC1 for complete general method for only one of odd or even eg $n = 2m$ leading to $2(4m^3 - m)$	3
7	(i) 1 (ii) 1000	2 1	B1 for 5^0 or for 25 × 1/25 o.e.	3
8	(i) 2/3 www (ii) $43 - 30\sqrt{2}$ www as final answer	2 3	M1 for 4/6 or for $\sqrt{48} = 2\sqrt{12}$ or $4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$ or $\sqrt{108} = 3\sqrt{12}$ or for $\sqrt{\frac{4}{9}}$ M2 for 3 terms correct of $25 - 15\sqrt{2} - 15\sqrt{2} + 18$ soi, M1 for 2 terms correct	5
9	(i) $(x + 3)^2 - 4$ (ii) ft their $(-a, b)$; if error in (i), accept $(-3, -4)$ if evidence of being independently obtained	3 2	B1 for $a = 3$, B2 for $b = -4$ or M1 for $5 - 3^2$ soi B1 each coord.; allow $x = -3$, $y = -4$; or M1 for $\begin{bmatrix}-3\\-4\end{bmatrix}$ o.e. oe for sketch with −3 and −4 marked on axes but no coords given	5
10	$(x^2 - 9)(x^2 + 4)$ $x^2 = 9$ [or −4] or ft for integers /fractions if first M1 earned $x = \pm 3$ cao	M2 M1 A1	or correct use of quad formula or comp sq reaching 9 and −4; allow M1 for attempt with correct eqn at factorising with factors giving two terms correct, or sign error, or attempt at formula or comp sq [no more than two errors in formula/substn]; for this first M2 or M1 allow use of y etc or of x instead of x^2 must have x^2; or M1 for $(x + 3)(x - 3)$; this M1 may be implied by $x = \pm 3$ A0 if extra roots if M0 then allow SC1 for use of factor theorem to obtain both 3 and −3 as roots or $(x + 3)$ and $(x - 3)$ found as factors and SC2 for $x^2 + 4$ found as other factor using factor theorem [ie max SC3]	4
				20

Section B

11	**i**	$y = 3x$	2	M1 for grad AB = $\frac{1-3}{6}$ or − 1/3 o.e.	2
	ii	eqn AB is $y = -1/3\,x + 3$ o.e. or ft	M1	need not be simplified; no ft from midpt used in (i); may be seen in (i) but do not give mark unless used in (ii)	
		$3x = -1/3x + 3$ or ft $x = 9/10$ or 0.9 o.e. cao	M1 A1	eliminating *x* or *y* , ft their eqns if find *y* first, cao for *y* then ft for *x*	
		$y = 27/10$ oe ft their $3 \times$ their x	A1	ft dep on both Ms earned	4
	iii	$\left(\frac{9}{10}\right)^2(1+3^2)$ o.e and completion to given answer	2	or square root of this; M1 for $\left(\frac{9}{10}\right)^2+\left(\frac{27}{10}\right)^2$ or 0.81 + 7.29 soi or ft their coords (inc midpt) <u>or</u> M1 for distance = 3 cos θ and tan θ = 3 and M1 for showing $\sin\theta = \frac{3}{\sqrt{10}}$ and completion	2
	iv	$2\sqrt{10}$	2	M1 for $6^2 + 2^2$ or 40 or square roots of these	2
	v	9 www or ft their $a\sqrt{10}$	2	M1 for ½ × 3 × 6 or $\frac{1}{2}\times \text{their}\,2\sqrt{10}\times\frac{9}{10}\sqrt{10}$	2
					12

12	**iA**	expansion of one pair of brackets correct 6 term expansion	M1 M1	eg $[(x + 1)](x^2 - 6x + 8)$; need not be simplified eg $x^3 - 6x^2 + 8x + x^2 - 6x + 8$; or M2 for correct 8 term expansion: $x^3 - 4x^2 + x^2 - 2x^2 + 8x - 4x - 2x + 8$, M1 if one error allow equivalent marks working backwards to factorisation, by long division or factor theorem etc or M1 for all three roots checked by factor theorem and M1 for comparing coeffts of x^3	2
	iB	cubic the correct way up x-axis: −1, 2, 4 shown y-axis 8 shown	G1 G1 G1	with two tps and extending beyond the axes at 'ends' ignore a second graph which is a translation of the correct graph	3
	iC	$[y=](x - 2)(x - 5)(x - 7)$ isw or $(x - 3)^3 - 5(x - 3)^2 + 2(x - 3) + 8$ isw or $x^3 - 14x^2 + 59x - 70$ (0, −70) or $y = -70$	2 1	M1 if one slip or for $[y =]$ f$(x - 3)$ or for roots identified at 2, 5, 7 or for translation 3 to the left allow M1 for complete attempt: $(x + 4)(x + 1)(x - 1)$ isw or $(x + 3)^3 - 5(x + 3)^2 + 2(x + 3) + 8$ isw allow 1 for (0, −4) or $y = -4$ after f$(x + 3)$ used	3
	ii	$27 - 45 + 6 + 8 = -4$ or $27 - 45 + 6 + 12 = 0$ long division of f(x) or their f$(x) + 4$ by $(x - 3)$ attempted as far as $x^3 - 3x^2$ in working $x^2 - 2x - 4$ obtained $[x =]\dfrac{2 \pm \sqrt{(-2)^2 - 4\times(-4)}}{2}$ or $(x - 1)^2 = 5$ $\dfrac{2 \pm \sqrt{20}}{2}$ o.e. isw or $1 \pm \sqrt{5}$	B1 M1 A1 M1 A1	or correct long division of $x^3 - 5x^2 + 2x + 12$ by $(x - 3)$ with no remainder or of $x^3 - 5x^2 + 2x + 8$ with rem −4 or inspection with two terms correct eg $(x - 3)(x^2$ ……….$- 4)$ dep on previous M1 earned; for attempt at formula or comp square on their other 'factor'	5 13

13	i	(5, 2) $\sqrt{20}$ or $2\sqrt{5}$	1 1	0 for $\pm\sqrt{20}$ etc	2
	ii	no, since $\sqrt{20}$ < 5 or showing roots of $y^2 - 4y + 9 = 0$ o.e. are not real	2	or ft from their centre and radius M1 for attempt (no and mentioning $\sqrt{20}$ or 5) or sketch or solving by formula or comp sq $(-5)^2 + (y-2)^2 = 20$ [condone one error] or SC1 for fully comparing distance from *x* axis with radius and saying yes	2
	iii	$y = 2x - 8$ or simplified alternative	2	M1 for $y - 2 = 2(x - 5)$ or ft from **(i)** or M1 for $y = 2x + c$ and subst their **(i)** or M1 for ans $y = 2x + k$, $k \neq 0$ or -8	2
	iv	$(x - 5)^2 + (2x)^2 = 20$ o.e.	M1	subst $2x + 2$ for *y* [oe for *x*]	
		$5x^2 - 10x + 5[= 0]$ or better equiv.	M1	expanding brackets and rearranging to 0; condone one error; dep on first M1	
		obtaining $x = 1$ (with no other roots) or showing roots equal	M1		
		one intersection [so tangent]	A1	o.e.; must be explicit; or showing line joining (1,4) to centre is perp to $y = 2x + 2$	
		(1, 4) cao	A1	allow $y = 4$	
		<u>alt method</u> $y - 2 = -½ (x - 5)$ o.e. $2x + 2 - 2 = -½ (x - 5)$ o.e. $x = 1$ $y = 4$ cao showing (1, 4) is on circle	 M1 M1 A1 A1 B1	line through centre perp to $y = 2x + 2$ dep; subst to find intn with $y = 2x + 2$ by subst in circle eqn or finding dist from centre = $\sqrt{20}$ [a similar method earns first M1 for eqn of diameter, 2nd M1 for intn of diameter and circle A1 each for *x* and *y* coords and last B1 for showing (1, 4) on line – award onlyA1 if (1, 4) and (9, 0) found without (1, 4) being identified as the soln]	
		<u>alt method</u> perp dist between $y = 2x - 8$ and $y = 2x + 2 = 10 \cos\theta$ where $\tan\theta = 2$ showing this is $\sqrt{20}$ so tgt $x = 5 - \sqrt{20}\sin\theta$ $x = 1$ (1, 4) cao	 M1 M1 M1 A1 A1	 or other valid method for obtaining *x* allow $y = 4$	 5 11