



# GCE

## Mathematics (MEI)

Advanced Subsidiary GCE 4751

Introduction to Advanced Mathematics (C1)

## Mark Scheme for June 2010

---

## SECTION A

1	$y = 3x + c$ or $y - y_1 = 3(x - x_1)$  $y - 5 = \text{their } m(x - 4)$ o.e.  $y = 3x - 7$ or simplified equiv.	<b>M1</b>  <b>M1</b>  <b>A1</b>	allow M1 for 3 clearly stated/ used as gradient of required line  or (4, 5) subst in their $y = mx + c$ ; allow M1 for $y - 5 = m(x - 4)$ o.e.  condone $y = 3x + c$ and $c = -7$ or <b>B3</b> www
2	(i) $250a^6b^7$  (ii) 16 cao  (iii) 64	<b>2</b>  <b>1</b>  <b>2</b>	<b>B1</b> for two elements correct; condone multiplication signs left in SC1 for eg $250 + a^6 + b^7$  condone $\pm 64$ <b>M1</b> for $[\pm]4^3$ or for $\sqrt{4096}$ or for only $-64$
3	$ac = \sqrt{y} - 5$ o.e. $ac + 5 = \sqrt{y}$ o.e. $[y =](ac + 5)^2$ o.e. isw	<b>M1</b>  <b>M1</b>  <b>M1</b>	<b>M1</b> for each of 3 correct or ft correct steps s.o.i. leading to $y$ as subject  or some/all steps may be combined; allow <b>B3</b> for $[y =](ac + 5)^2$ o.e. isw or <b>B2</b> if one error
4 (i)	$2 - 2x > 6x + 5$  $-3 > 8x$ o.e. or ft  $x < -3/8$ o.e. or ft isw	<b>M1</b>  <b>M1</b>  <b>M1</b>	or $1 - x > 3x + 2.5$  for collecting terms of their inequality correctly on opposite sides eg $-8x > 3$ allow <b>B3</b> for correct inequality found after working with equation allow <b>SC2</b> for $-3/8$ o.e. found with equation or wrong inequality
4 (ii)	$-4 < x < 1/2$ o.e.	<b>2</b>	accept as two inequalities <b>M1</b> for one 'end' correct or for $-4$ and $1/2$
5 (i)	$7\sqrt{3}$	<b>2</b>	<b>M1</b> for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$

4751

Mark Scheme

June 2010

5 (ii)	$\frac{10+15\sqrt{2}}{7}$ www isw	3	<b>B1</b> for 7 [B0 for 7 wrongly obtained]  <u>and B2</u> for $10+15\sqrt{2}$ or <b>B1</b> for one term of numerator correct;  if <b>B0</b> , then <b>M1</b> for attempt to multiply num and denom by $3+\sqrt{2}$
6	$5 + 2k$ soi  $k = 12$  attempt at $f(3)$  $27 + 36 + m = 59$ o.e.  $m = -4$ cao	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>	allow <b>M1</b> for expansion with $5x^3 + 2kx^3$ and no other $x^3$ terms or <b>M1</b> for $(29 - 5) / 2$ soi  must substitute 3 for $x$ in cubic not product or long division as far as obtaining $x^2 + 3x$ in quotient  or from division $m - (-63) = 59$ o.e. or for $27 + 3k + m = 59$ or ft their $k$
7	$1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$ oe (must be simplified) isw	4	<b>B3</b> for 4 terms correct, or <b>B2</b> for 3 terms correct or for all correct but unsimplified (may be at an earlier stage, but factorial or ${}^nC_r$ notation must be expanded/worked out) or <b>B1</b> for 1, 4, 6, 4, 1 soi or for $1 + \dots + \frac{1}{16}x^4$ [must have at least one other term]
8	$5(x + 2)^2 - 14$	4	<b>B1</b> for $a = 5$ , <u>and B1</u> for $b = 2$ <u>and B2</u> for $c = -14$ or <b>M1</b> for $c = 6$ - their $ab^2$ or <b>M1</b> for [their $a$ ](6/their $a$ - their $b^2$ ) [no ft for $a = 1$ ]
9	mention of $-5$ as a square root of 25 or $(-5)^2 = 25$  $-5 - 5 \neq 0$ o.e. or $x + 5 = 0$	<b>M1</b>  <b>M1</b>	condone $-5^2 = 25$  or, dep on first <b>M1</b> being obtained, allow <b>M1</b> for showing that 5 is the only soln of $x - 5 = 0$  allow <b>M2</b> for $x^2 - 25 = 0$ $(x + 5)(x - 5) [= 0]$ so $x - 5 = 0$ or $x + 5 = 0$

Section A Total: 36

## SECTION B

<b>10 (i)</b>	$(2x - 3)(x + 1)$  $x = 3/2$ and $-1$ obtained	<b>M2</b>  <b>B1</b>	<b>M1</b> for factors with one sign error or giving two terms correct allow <b>M1</b> for $2(x - 1.5)(x + 1)$ with no better factors seen  or ft their factors
<b>10 (ii)</b>	graph of quadratic the correct way up and crossing both axes  crossing $x$ -axis only at $3/2$ and $-1$ or ft from their roots in (i), or their factors if roots not given  crossing $y$ -axis at $-3$	<b>B1</b>  <b>B1</b>  <b>B1</b>	for $x = 3/2$ condone 1 and 2 marked on axis and crossing roughly halfway between; intns must be shown labelled or worked out nearby
<b>10 (iii)</b>	use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula)  $25 - 40 < 0$ or $-15$ obtained	<b>M1</b>  <b>A1</b>	may be in formula or $(x - 2.5)^2 = 6.25 - 10$ or $(x - 2.5)^2 + 3.75 = 0$ oe (condone one error)  or $\sqrt{-15}$ seen in formula or $(x - 2.5)^2 = -3.75$ oe or $x = 2.5 \pm \sqrt{-3.75}$ oe
<b>10 (iv)</b>	$2x^2 - x - 3 = x^2 - 5x + 10$ o.e.  $x^2 + 4x - 13 [= 0]$  use of quad. formula on resulting eqn (do not allow for original quadratics used)  $-2 \pm \sqrt{17}$ cao	<b>M1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>	attempt at eliminating $y$ by subst or subtraction  or $(x + 2)^2 = 17$ ; for rearranging to form $ax^2 + bx + c [= 0]$ or to completing square form condone one error for each of 2 <sup>nd</sup> and 3 <sup>rd</sup> <b>M1s</b>  or $x + 2 = \pm\sqrt{17}$ o.e. 2 <sup>nd</sup> and 3 <sup>rd</sup> <b>M1s</b> may be earned for good attempt at completing square as far as roots obtained

<p><b>11 (i)</b></p>	<p>grad AB = <math>\frac{1-3}{5-(-1)}</math> [= -1/3]  <math>y - 3 =</math> their grad <math>(x - (-1))</math> or  <math>y - 1 =</math> their grad <math>(x - 5)</math></p> <p><math>y = -1/3x + 8/3</math> or <math>3y = -x + 8</math> o.e  isw</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>or use of <math>y =</math> their gradient <math>x + c</math>  with coords of A or B  or <b>M2</b> for <math>\frac{y-3}{1-3} = \frac{x-(-1)}{5-(-1)}</math> o.e.</p> <p>o.e. eg <math>x + 3y - 8 = 0</math> or <math>6y = 16 - 2x</math>  allow <b>B3</b> for correct eqn www</p>
<p><b>11 (ii)</b></p>	<p>when <math>y = 0, x = 8</math>; when <math>x = 0,</math>  <math>y = 8/3</math> or ft their (i)</p> <p>[Area =] <math>\frac{1}{2} \times 8/3 \times 8</math> o.e. cao isw</p>	<p><b>M1</b></p> <p><b>M1</b></p>	<p>allow <math>y = 8/3</math> used without  explanation if already seen in eqn in  (i)</p> <p><b>M1</b> NB answer <math>32/3</math> given;  allow <math>4 \times 8/3</math> if first <b>M1</b> earned;  or  <b>M1</b> for  <math>\int_0^8 \left[ \frac{1}{3}(8-x) \right] dx = \left[ \frac{1}{3} \left( 8x - \frac{1}{2}x^2 \right) \right]_0^8</math>  and <b>M1</b> dep for <math>\frac{1}{3}(64 - 32[-0])</math></p>
<p><b>11 (iii)</b></p>	<p>grad perp = <math>-1/\text{grad AB}</math> stated, or  used after their grad AB stated in  this part</p> <p>midpoint [of AB] = (2, 2)</p> <p><math>y - 2 =</math> their grad perp <math>(x - 2)</math> or ft  their midpoint</p> <p><u>alt method working back from  ans:</u></p> <p>grad perp = <math>-1/\text{grad AB}</math> and  showing/stating same as given  line</p> <p>finding intn of their  <math>y = -1/3x - 8/3</math> and <math>y = 3x - 4</math> is  (2, 2)</p> <p>showing midpt of AB is (2, 2)</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>or</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p>	<p>or showing <math>3 \times -1/3 = -1</math>  if (i) is wrong, allow the first <b>M1</b>  here ft, provided the answer is  correct ft</p> <p>must state ‘midpoint’ or show  working</p> <p>for <b>M3</b> this must be correct, starting  from grad AB = <math>-1/3</math>, and also  needs correct completion to given  ans <math>y = 3x - 4</math></p> <p>mark one method or the other, to  benefit of candidate, not a mixture</p> <p>eg stating <math>-1/3 \times 3 = -1</math></p> <p>or showing that (2, 2) is on <math>y = 3x - 4,</math>  having found (2, 2) first</p> <p>[for both methods: for <b>M3</b> must be  fully correct]</p>

4751

## Mark Scheme

June 2010

<b>11 (iv)</b>	subst $x = 3$ into $y = 3x - 4$ and obtaining centre = (3, 5)  $r^2 = (5 - 3)^2 + (1 - 5)^2$ o.e.  $r = \sqrt{20}$ o.e. cao  eqn is $(x - 3)^2 + (y - 5)^2 = 20$ or ft their $r$ and $y$ -coord of centre	<b>M1</b>  <b>M1</b>  <b>A1</b>  <b>B1</b>	or using $(-1 - 3)^2 + (3 - b)^2 = (5 - 3)^2 + (1 - b)^2$ and finding (3, 5)  or $(-1 - 3)^2 + (3 - 5)^2$ or ft their centre using A or B  condone $(x - 3)^2 + (y - b)^2 = r^2$ o.e. or $(x - 3)^2 + (y - \text{their } 5)^2 = r^2$ o.e. (may be seen earlier)
<b>12 (i)</b>	trials of at calculating $f(x)$ for at least one factor of 30  details of calculation for $f(2)$ or $f(-3)$ or $f(-5)$  attempt at division by $(x - 2)$ as far as $x^3 - 2x^2$ in working  correctly obtaining $x^2 + 8x + 15$  factorising a correct quadratic factor  $(x - 2)(x + 3)(x + 5)$	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>	<b>M0</b> for division or inspection used    or equiv for $(x + 3)$ or $(x + 5)$ ; or inspection with at least two terms of quadratic factor correct or B2 for another factor found by factor theorem  for factors giving two terms of quadratic correct; M0 for formula without factors found  condone omission of first factor found; ignore '= 0' seen  allow last four marks for $(x - 2)(x + 3)(x + 5)$ obtained; for all 6 marks must see factor theorem use first
<b>12 (ii)</b>	sketch of cubic right way up, with two turning points  values of intns on $x$ axis shown, correct $(-5, -3, \text{ and } 2)$ or ft from their factors/ roots in (i)  $y$ -axis intersection at $-30$	<b>B1</b>  <b>B1</b>  <b>B1</b>	0 if stops at $x$ -axis  on graph or nearby in this part  mark intent for intersections with both axes  or $x = 0, y = -30$ seen in this part if consistent with graph drawn

4751

Mark Scheme

June 2010

12	(iii)	<p><math>(x - 1)</math> substituted for <math>x</math> in either form of eqn for <math>y = f(x)</math></p> <p><math>(x - 1)^3</math> expanded correctly (need not be simplified) or two of their factors multiplied correctly</p> <p>correct completion to given answer [condone omission of 'y =']</p>	<b>M1</b>	<p>correct or ft their (i) or (ii) for factorised form; condone one error; allow for new roots stated as <math>-4, -2</math> and <math>3</math> or ft</p>
			<b>M1 dep</b>	<p>or <b>M1</b> for correct or correct ft multiplying out of all 3 brackets at once, condoning one error [<math>x^3 - 3x^2 + 4x^2 + 2x^2 + 8x - 6x - 12x - 24</math>]</p>
			<b>M1</b>	<p>unless all 3 brackets already expanded, must show at least one further interim step allow <b>SC1</b> for <math>(x + 1)</math> subst <u>and</u> correct exp of <math>(x + 1)^3</math> or two of their factors ft</p> <p><u>or</u>, for those using given answer: <b>M1</b> for roots stated or used as <math>-4, -2</math> and <math>3</math> or ft <b>A1</b> for showing all 3 roots satisfy given eqn <b>B1</b> for comment re coefft of <math>x^3</math> or product of roots to show that eqn of translated graph is not a multiple of RHS of given eqn</p>

Section B Total: 36