GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

SECTION A

1	$x > -13/4$ o.e. isw www	**3**	condone $x > 13/-4$ or $13/-4 < x$; **M2** for $4x > -13$ or **M1** for one side of this correct with correct inequality, and **B1** for final step ft from their $ax > b$ or $c > dx$ for $a \neq 1$and $d \neq 1$; if no working shown, allow **SC1** for $-13/4$ oe with equals sign or wrong inequality	**M1** for $13 > -4x$ (may be followed by $13/-4 > x$, which earns no further credit); $6x + 3 > 2x + 5$ is an error not an MR; can get **M1** for $4x >$ ... following this, and then a possible **B1**
2	7	**2**	condone $y = 7$ or (5, 7); **M1** for $\frac{k-(-5)}{5-1} = 3$ or other correct use of gradient eg triangle with 4 across, 12 up	condone omission of brackets; or **M1** for correct method for eqn of line and $x = 5$ subst in their eqn and evaluated to find k; or **M1** for both of $y - k = 3(x - 5)$ oe and $y - (-5) = 3(x - 1)$ oe
3	(i) 4/3 isw	**2**	condone ±4/3; **M1** for numerator or denominator correct or for $\frac{3}{4}$ or $\frac{1}{\left(\frac{3}{4}\right)}$ oe or for $\left(\frac{16}{9}\right)^{\frac{1}{2}}$ soi	**M1** for just $-4/3$; allow **M1** for $\sqrt{16} = 4$ and $\sqrt{9} = 3$ soi; condone missing brackets

3	(ii) $\frac{2a}{c^5}$ or $2ac^{-5}$	**3**	**B1** for each 'term' correct; mark final answer; if B0, then **SC1** for $(2ac^2)^3 = 8a^3c^6$ or $72a^5c^7$ seen	condone a^1; condone multiplication signs but **0** for addition signs
4	(i) (10, 4)	**2**	**0** for (5, 4); otherwise **1** for each coordinate	ignore accompanying working / description of transformation; condone omission of brackets; (Image includes back page for examiners to check that there is no work there)
4	(ii) (5, 11)	**2**	**0** for (5, 4); otherwise **1** for each coordinate	ignore accompanying working / description of transformation; condone omission of brackets
5	6000	**4**	**M3** for $15 \times 5^2 \times 2^4$; or **M2** for two of these elements correct with multiplication or all three elements correct but without multiplication (e.g. in list or with addition signs); or **M1** for 15 soi or for 1 6 15 ... seen in Pascal's triangle; **SC2** for 20000[x^3]	condone inclusion of x^4 eg $(2x)^4$; condone omission of brackets in $2x^4$ if 16 used; allow **M3** for correct term seen (often all terms written down) but then wrong term evaluated or all evaluated and correct term not identified; $15 \times 5^2 \times (2x)^4$ earns **M3** even if followed by $15 \times 25 \times 2$ calculated; no MR for wrong power evaluated but **SC** for fourth term evaluated

6	$2x^3 + 9x^2 + 4x - 15$	**3**	as final answer; ignore '= 0'; **B2** for 3 correct terms of answer seen or for an 8-term or 6 term expansion with at most one error: or **M1** for correct quadratic expansion of one pair of brackets; or **SC1** for a quadratic expansion with one error then a good attempt to multiply by the remaining bracket	correct 8-term expansion: $2x^3 + 6x^2 - 2x^2 + 5x^2 - 6x + 15x - 5x - 15$ correct 6-term expansions: $2x^3 + 4x^2 + 5x^2 - 6x + 10x - 15$ $2x^3 + 6x^2 + 3x^2 + 9x - 5x - 15$ $2x^3 + 11x^2 - 2x^2 + 15x - 11x - 15$ for **M1**, need not be simplified; ie **SC1** for knowing what to do and making a reasonable attempt, even if an error at an early stage means more marks not available
7	$b^2 - 4ac$ soi	**M1**		allow seen in formula; need not have numbers substituted but discriminant part must be correct;
	1 www	**A1**	or **B2**	clearly found as discriminant, or stated as $b^2 - 4ac$, not just seen in formula eg **M1A0** for $\sqrt{b^2 - 4ac} = \sqrt{1} = 1$;
	2 [distinct real roots]	**B1**	**B0** for finding the roots but not saying how many there are	condone discriminant not used; ignore incorrect roots found

8	$yx + 3y = 1 - 2x$ oe or ft	**M1**	for multiplying to eliminate denominator <u>and</u> for expanding brackets, or for correct division by y <u>and</u> writing as separate fractions: $x+3=\frac{1}{y}-\frac{2x}{y}$;	each mark is for carrying out the operation correctly; ft earlier errors for equivalent steps if error does not simplify problem; some common errors:
	$yx + 2x = 1 - 3y$ oe or ft	**M1**	for collecting terms; dep on having an ax term and an xy term, oe after division by y,	$y\,(x + 3) = 1 - 2x$ $yx + 3x = 1 - 2x$ **M0** $yx + 5x = 1$ **M1** ft $x\,(y + 5) = 1$ **M1** ft $x=\frac{1}{y+5}$ **M1** ft $yx + 3 = 1 - 2x$ **M0** $yx + 2x = -2$ **M1** ft $x\,(y + 2) = -2$ **M1** ft $x=\frac{-2}{y+2}$ **M1** ft
	$x\,(y + 2) = 1 - 3y$ oe or ft	**M1**	for taking out x factor; dep on having an ax term and an xy term, oe after division by y,	
	$[x=]\frac{1-3y}{y+2}$ oe or ft as final answer	**M1**	for division with no wrong work after; dep on dividing by a two-term expression; last M not earned for triple-decker fraction as final answer	for **M4**, must be completely correct;

9	$x + 2y = k$ ($k \neq 6$) or $y = -\frac{1}{2}x + c$ ($c \neq 3$)	**M1**	for attempt to use gradients of parallel lines the same; **M0** if just given line used;	eg following an error in manipulation, getting original line as $y = \frac{1}{2}x + 3$ then using $y = \frac{1}{2}x + c$ earns **M1** and can then go on to get **A0** for $y = \frac{1}{2}x - 4$, **M1** for (0, −4) **M1** for (8, 0) and **A0** for area of 16;
	$x + 2y = 12$ or [$y =$]$-\frac{1}{2}x + 6$ oe	**A1**	or **B2**; must be simplified; or evidence of correct 'stepping' using (10, 1) eg may be on diagram;	allow bod **B2** for a candidate who goes straight to $y = -\frac{1}{2}x + 6$ from $2y = -x + 6$; NB the equation of the line is not required; correct intercepts obtained will imply this **A1;**
	(12, 0) or ft	**M1**	or 'when $y = 0$, $x = 12$' etc or using 12 or ft as a limit of integration; intersections must ft from their line or 'stepping' diagram using their gradient	NB for intersections with axes, if both Ms are not gained, it must be clear which coord is being found eg **M0** for intn with x axis = 6 from correct eqn;; if the intersections are not explicit, they may be implied by the area calculation eg use of ht = 6 or the correct ft area found;
	(0, 6)or ft	**M1**	or integrating to give $-\frac{1}{4}x^2 + 6x$ or ft their line	allow ft from the given line as well as others for both these intersection Ms;
	36 [sq units] cao	**A1**	or **B3** www	NB **A0** if 36 is incorrectly obtained eg after intersection $x = -12$ seen (which earns **M0** from correct line);

10	$n(n+1)(n+2)$ argument from general consecutive numbers leading to: at least one must be even [exactly] one must be multiple of 3	**M1** **A1** **A1**	condone division by n and then $(n+1)(n+2)$ seen, or separate factors shown after factor theorem used; or divisible by 2; if M0: allow **SC1** for showing given expression always even	ignore ‘ = 0’; an induction approach using the factors may also be used eg by those doing paper FP1 as well; **A0** for just substituting numbers for n and stating results; allow **SC2** for a correct induction approach using the original cubic (**SC1** for each of showing even and showing divisible by 3)

SECTION B

11	(i) $x + 4x^2 + 24x + 31 = 10$ oe $4x^2 + 25x + 21$ [= 0] $(4x + 21)(x + 1)$ $x = -1$ or $-21/4$ oe isw $y = 11$ or $61/4$ oe isw	**M1** **M1** **M1** **A1** **A1**	for subst of x or y or subtraction to eliminate variable; condone one error; for collection of terms and rearrangement to zero; condone one error; for factors giving at least two terms of their quadratic correct or for subst into formula with no more than two errors [dependent on attempt to rearrange to zero]; or **A1** for $(-1, 11)$ and **A1** for $(-21/4, 61/4)$ oe	or $4y^2 - 105y + 671$ [= 0]; eg condone spurious $y = 4x^2 + 25x + 21$ as one error (and then count as eligible for 3rd **M1**); or $(y - 11)(4y - 61)$; [for full use of completing square with no more than two errors allow 2nd and 3rd **M1**s simultaneously]; from formula: accept $x = -1$ or $-42/8$ oe isw
11	(ii) $4(x + 3)^2 - 5$ isw	**4**	**B1** for $a = 4$, **B1** for $b = 3$, **B2** for $c = -5$ or **M1** for $31 - 4 \times$ their b^2 soi or for $-5/4$ or for $31/4 -$ their b^2 soi	eg an answer of $(x + 3)^2 - {}^5/_4$ earns **B0 B1 M1**; $1(2x + 6)^2 - 5$ earns **B0 B0 B2**; 4(earns first **B1**; condone omission of square symbol
11	(iii)(*A*) $x = -3$ or ft ($-$their b) from (ii)	**1**		**0** for just -3 or ft; **0** for $x = -3$, $y = -5$ or ft
11	(iii)(*B*) -5 or ft their c from (ii)	**1**	allow $y = -5$ or ft	**0** for just $(-3, -5)$; bod **1** for $x = -3$ stated then $y = -5$ or ft

12	(i) $y = 2x + 5$ drawn -2, -1.4 to -1.2, 0.7 to 0.85	**M1** **A2**	 **A1** for two of these correct	condone unruled and some doubling; tolerance: must pass within/touch at least two circles on overlay; the line must be drawn long enough to intersect curve at least twice; condone coordinates or factors
12	(ii) $4 = 2x^3 + 5x^2$ or $2x + 5 - \frac{4}{x^2} = 0$ and completion to given answer $f(-2) = -16 + 20 - 4 = 0$ use of $x + 2$ as factor in long division of given cubic as far as $2x^3 + 4x^2$ in working $2x^2 + x - 2$ obtained $[x =]\frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times -2}}{2 \times 2}$ oe $\frac{-1 \pm \sqrt{17}}{4}$ oe isw	**B1** **B1** **M1** **A1** **M1** **A1**	 or correct division / inspection showing that $x + 2$ is factor; or inspection or equating coefficients, with at least two terms correct; dep on previous M1 earned; for attempt at formula or full attempt at completing square, using their other factor	condone omission of final '= 0'; may be set out in grid format condone omission of + sign (eg in grid format) not more than two errors in formula / substitution / completing square; allow even if their 'factor' has a remainder shown in working; **M0** for just an attempt to factorise

12	(iii) $\frac{4}{x^2} = x + 2$ or $y = x + 2$ soi $y = x + 2$ drawn 1 real root	**M1** **A1** **A1**	eg is earned by correct line drawn	condone intent for line; allow slightly out of tolerance; condone unruled; need drawn for $-1.5 \le x \le 1.2$; to pass through/touch relevant circle(s) on overlay
13	(i) [radius =] 4 [centre] (4, 2)	**B1** **B1**	**B0** for ± 4	 condone omission of brackets

13	(ii) $(x-4)^2 + (-2)^2 = 16$ oe	**M1**	for subst $y = 0$ in circle eqn;	NB candidates may expand and rearrange eqn first, making errors – they can still earn this **M1** when they subst $y = 0$ in their circle eqn; condone omission of $(-2)^2$ for this first **M1** only; not for second and third **M1**s; do not allow substitution of $x = 0$ for any Ms in this part
	$(x-4)^2 = 12$ or $x^2 - 8x + 4$ [= 0]	**M1**	putting in form ready to solve by comp sq, or for rearrangement to zero; condone one error;	eg allow **M1** for $x^2 + 4 = 0$ [but this two-term quadratic is not eligible for 3rd **M1**];
	$x-4 = \pm\sqrt{12}$ or $[x =]\dfrac{8 \pm \sqrt{8^2 - 4\times1\times4}}{2\times1}$	**M1**	for attempt at comp square or formula; dep on previous M2 earned and on three-term quadratic;	not more than two errors in formula / substitution; allow **M1** for $x-4=\sqrt{12}$; **M0** for just an attempt to factorise
	$[x =]4 \pm \sqrt{12}$ or $4 \pm 2\sqrt{3}$ or $\dfrac{8 \pm \sqrt{48}}{2}$ oe isw	**A1**		
	or	**or**		
	sketch showing centre (4, 2) and triangle with hyp 4 and ht 2	**M1**		
	$4^2 - 2^2 = 12$	**M1**	or the square root of this; implies previous M1 if no sketch seen;	
	$[x =]4 \pm \sqrt{12}$ oe	**A2**	**A1** for one solution	

13	(iii) subst $\left(4+2\sqrt{2},\ 2+2\sqrt{2}\right)$ into circle eqn and showing at least one step in correct completion	**B1**	or showing sketch of centre C and A and using Pythag: $\left(2\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2=8+8=16$;	or subst the value for one coord in circle eqn and correctly working out the other as a possible value;
	Sketch of both tangents	**M1**		need not be ruled; must have negative gradients with tangents intended to be parallel and one touching above and to right of centre; mark intent to touch – allow just missing or just crossing circle twice; condone A not labelled
	grad tgt = −1 or −1/their grad CA	**M1**	allow ft after correct method seen for grad CA = $\dfrac{2+2\sqrt{2}-2}{4+2\sqrt{2}-4}$ oe (may be on/ near sketch);	allow ft from wrong centre found in (i);
	$y-(2+2\sqrt{2})=\text{their } m(x-(4+2\sqrt{2}))$	**M1**	or y = their $mx + c$ and subst of $\left(4+2\sqrt{2},\ 2+2\sqrt{2}\right)$;	for intent; condone lack of brackets for **M1**; independent of previous Ms; condone grad of CA used;
	$y=-x+6+4\sqrt{2}$ oe isw	**A1**	accept simplified equivs eg $x+y=6+4\sqrt{2}$;	**A0** if obtained as eqn of other tangent instead of the tangent at A (eg after omission of brackets);
	parallel tgt goes through $\left(4-2\sqrt{2},\ 2-2\sqrt{2}\right)$	**M1**	or ft wrong centre; may be shown on diagram; may be implied by correct equation for the tangent (allow ft their gradient);	no bod for just $y-2-2\sqrt{2}=-1(x-4-2\sqrt{2})$ without first seeing correct coordinates;
	eqn is $y=-x+6-4\sqrt{2}$ oe isw	**A1**	accept simplified equivs eg $x+y=6-4\sqrt{2}$	**A0** if this is given as eqn of the tangent at A instead of other tangent (eg after omission of brackets)

Section B Total: 36

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553