6.2 CONCEPTS FOR ADVANCED MATHEMATICS, C2 (4752) AS

Objectives

To introduce students to a number of topics which are fundamental to the advanced study of mathematics.

Assessment

Examination (72 marks)
1 hour 30 minutes.
The examination paper has two sections.

Section A: 8-10 questions, each worth no more than 5 marks.
Section Total: 36 marks

Section B: three questions, each worth about 12 marks.
Section Total: 36 marks

Assumed Knowledge

Candidates are expected to know the content of Intermediate Tier GCSE* and *C1*.

*See note on page 34.

Subject Criteria

The Units *C1* and *C2* are required for Advanced Subsidiary GCE Mathematics in order to ensure coverage of the subject criteria.

The Units *C1, C2, C3* and *C4* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

CONCEPTS FOR ADVANCED MATHEMATICS, C2		
Specification	**Ref.**	**Competence Statements**

ALGEBRA		
Logarithms.	C2a1	Understand the meaning of the word logarithm.
	2	Understand the laws of logarithms and how to apply them.
	3	Know the values of $\log_a a$ and $\log_a 1$.
	4	Know how to convert from an index to a logarithmic form and vice versa.
	5	Know the function $y = a^x$ and its graph.
	6	Be able to solve an equation of the form $a^x = b$.
	7	Know how to reduce the equations $y = ax^n$ and $y = ab^x$ to linear form and, using experimental data, to draw a graph to find values of a, n and a, b.

SEQUENCES AND SERIES		
Definitions of sequences.	C2s1	Know what a sequence of numbers is and the meaning of finite and infinite sequences.
	2	Know that a sequence can be generated using a formula for the k^{th} term, or a recurrence relation of the form $a_{k+1} = \mathrm{f}(a_k)$.
	3	Know what a series is.
	4	Be familiar with $\sum$ notation.
	5	Know and be able to recognise the periodicity of sequences.
	6	Know the difference between convergent and divergent sequences.
Arithmetic series.	7	Know what is meant by arithmetic series and sequences.
	8	Be able to use the standard formulae associated with arithmetic series and sequences.
Geometric series.	9	Know what is meant by geometric series and sequences.
	10	Be able to use the standard formulae associated with geometric series and sequences.
	11	Know the condition for a geometric series to be convergent and be able to find its sum to infinity.
	12	Be able to solve problems involving arithmetic and geometric series and sequences.

CONCEPTS FOR ADVANCED MATHEMATICS, C2			
Ref.	**Notes**	**Notation**	**Exclusions**

ALGEBRA

Ref.	Notes	Notation	Exclusions
C2a1	$y = \log_a x \Leftrightarrow a^y = x$		
2	$\log_a (xy) = \log_a x + \log_a y$ $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$ $\log_a (x^k) = k \log_a x$		Change of base of logarithms.
3	$\log_a a = 1$, $\log_a 1 = 0$		
4	$x = a^n \Leftrightarrow n = \log_a x$		
5	For $a \geq 1$.		
6	By taking logarithms of both sides.		
7	By taking logarithms of both sides and comparing with the equation $y = mx + c$.		

SEQUENCES AND SERIES

Ref.	Notes	Notation	Exclusions
C2s1			
2	e.g. $a_k = 2 + 3k$; $a_{k+1} = a_k + 3$ with $a_1 = 5$.	k^{th} term: a_k	
3	With reference to the corresponding sequence.		
4	Including the sum of the first n natural numbers.		
5			
6	e.g. convergent sequence $a_k = 3 - \frac{1}{k}$ e.g. divergent sequence $a_k = 1 + 2k^2$		Formal tests for convergence.
7	The term arithmetic progression (AP) may also be used.	1st term, a Last term, l Common difference, d.	
8	The nth term, the sum to n terms.		
9	The term geometric progression (GP) may also be used.	1st term, a Common ratio, r.	
10	The nth term, the sum to n terms.	S_n	
11	Candidates will be expected to be familiar with the modulus sign in the condition for convergence.	$S_\infty = \frac{a}{1-r}$, $\lvert r \rvert < 1$	
12	These may involve the solution of quadratic and simultaneous equations.		

CONCEPTS FOR ADVANCED MATHEMATICS, C2		
Specification	**Ref.**	**Competence Statements**

TRIGONOMETRY		
Basic trigonometry.	C2t1	* Know how to solve right-angled triangles using trigonometry.
The sine, cosine and tangent functions.	2	Be able to use the definitions of sin θ and cos θ for any angle.
	3	Know the graphs of sin θ, cos θ and tan θ for all values of θ, their symmetries and periodicities.
	4	Know the values of sin θ, cos θ and tan θ when θ is 0°, 30°, 45°, 60°, 90° and 180°.
Identities.	5	Be able to use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (for any angle).
	6	Be able to use the identity $\sin^2\theta + \cos^2\theta = 1$.
	7	Be able to solve simple trigonometric equations in given intervals.
Area of a triangle.	8	Know and be able to use the fact that the area of a triangle is given by $\frac{1}{2}ab\sin C$.
The sine and cosine rules.	9	Know and be able to use the sine and cosine rules.
Radians.	10	Understand the definition of a radian and be able to convert between radians and degrees.
	11	Know and be able to find the arc length and area of a sector of a circle, when the angle is given in radians.

CONCEPTS FOR ADVANCED MATHEMATICS, C2			
Ref.	**Notes**	**Notation**	**Exclusions**

TRIGONOMETRY			
C2t1			
2	e.g. by reference to the unit circle.		
3	Their use to find angles outside the first quadrant.		
4	Exact values may be expected.		
5	e.g. solve $\sin\theta = 3\cos\theta$ for $0° \le \theta \le 360°$.		
6	e.g. simple application to solution of equations.		
7	e.g. $\sin\theta = 0.5 \Leftrightarrow \theta = 30°$, 150^{o} in $[0^{o}, 360^{o}]$.	arcsin *x* arccos *x* arctan *x*	Principal values (see C4) General solutions.
8			
9	Use of bearings may be required.		
10			
11	The results $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ where θ is measured in radians.		

CONCEPTS FOR ADVANCED MATHEMATICS, C2		
Specification	**Ref.**	**Competence Statements**

CALCULUS		
The basic process of differentiation.	C2c1	Know that the gradient of a curve at a point is given by the gradient of the tangent at the point.
	2	Know that the gradient of the tangent is given by the limit of the gradient of a chord.
	3	Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x.
Applications of differentiation to the graphs of functions.	4	Be able to differentiate $y = kx^n$ where k is a constant, and the sum of such functions.
	5	Be able to find second derivatives.
	6	Be able to use differentiation to find stationary points on a curve: maxima, minima and points of inflection.
	7	Understand the terms increasing function and decreasing function.
	8	Be able to find the equation of a tangent and normal at any point on a curve.
Integration as the inverse of differentiation.	9	Know that integration is the inverse of differentiation.
	10	Be able to integrate functions of the form kx^n where k is a constant and $n \neq -1$, and the sum of such functions.
	11	Know what are meant by indefinite and definite integrals.
	12	Be able to evaluate definite integrals.
	13	Be able to find a constant of integration given relevant information.
Integration to find the area under a curve.	14	Know that the area under a graph can be found as the limit of a sum of areas of rectangles.
	15	Be able to use integration to find the area between a graph and the x-axis.
	16	Be able to find an approximate value of a definite integral using the trapezium rule, and comment sensibly on its accuracy.

CURVE SKETCHING		
Stationary points.	C2C1	Be able to use stationary points when curve sketching.
Stretches.	2	Know how to sketch curves of the form $y = a\mathrm{f}(x)$ and $y = \mathrm{f}(ax)$, given the curve of: $y = \mathrm{f}(x)$.

CONCEPTS FOR ADVANCED MATHEMATICS, C2			
Ref.	**Notes**	**Notation**	**Exclusions**

CALCULUS

Ref.	Notes	Notation	Exclusions
C2c1			
2		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$	
3	The terms increasing function and decreasing function.		
4	Simple cases of differentiation from first principles. Including rational values of *n*.	$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$	
5		$f''(x) = \frac{d^2y}{dx^2}$	
6			
7	In relation to the sign of $\frac{dy}{dx}$.		
8			
9			
10			
11			
12	e.g. $\int_1^3 (3x^2 + 5x - 1)dx$.		
13	e.g. Find *y* when $x = 2$ given that $\frac{dy}{dx} = 2x + 5$ and $y = 7$ when $x = 1$.		
14	General understanding only.		Formal proof.
15	Includes areas of regions partly above and partly below the *x*-axis.		
16	Comments on the error will be restricted to consideration of its direction and made with reference to the shape of the curve.		Repeated applications of the trapezium rule (see C4).

CURVE SKETCHING

Ref.	Notes	Notation	Exclusions
C2C1	Including distinguishing between them.		
2	Simple cases only e.g. Given $f(x) = \sin x$, sketch $y = \sin(2x)$ or $y = 3\sin x$.		Combined transformations (see C3f2).