

ADVANCED GCE

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

4753/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$. [3]

2 A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

(i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find A and k . [5]

(ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. [3]

3 Sketch the curve $y = 2 \arccos x$ for $-1 \leq x \leq 1$. [3]

4 Fig. 4 shows a sketch of the graph of $y = 2|x - 1|$. It meets the x - and y -axes at $(a, 0)$ and $(0, b)$ respectively.

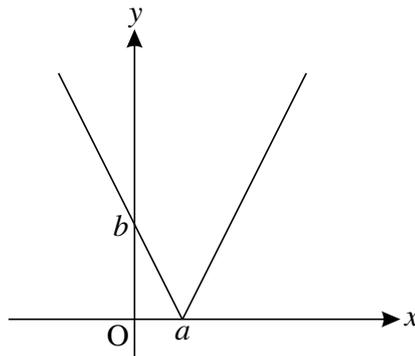


Fig. 4

Find the values of a and b . [3]

5 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y . [3]

(ii) Find an expression for y in terms of x , and differentiate it to verify the result in part (i). [4]

6 Given that $f(x) = \frac{x+1}{x-1}$, show that $ff(x) = x$.

Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? [5]

7 (i) Show that

$$(A) (x - y)(x^2 + xy + y^2) = x^3 - y^3,$$

$$(B) \left(x + \frac{1}{2}y\right)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2. \quad [4]$$

(ii) Hence prove that, for all real numbers x and y , if $x > y$ then $x^3 > y^3$. [3]

Section B (36 marks)

8 Fig. 8 shows the line $y = x$ and parts of the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

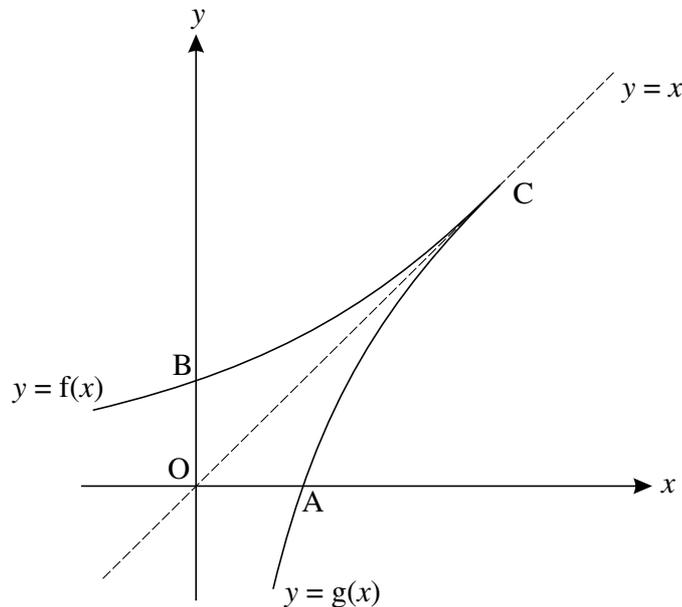


Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). [5]

(ii) Prove algebraically that $g(x)$ is the inverse of $f(x)$. [2]

(iii) Evaluate $\int_0^1 f(x) dx$, giving your answer in terms of e . [3]

(iv) Use integration by parts to find $\int \ln x dx$.

Hence show that $\int_{e^{-1}}^1 g(x) dx = \frac{1}{e}$. [6]

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. [2]

- 9 Fig. 9 shows the curve $y = \frac{x^2}{3x-1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

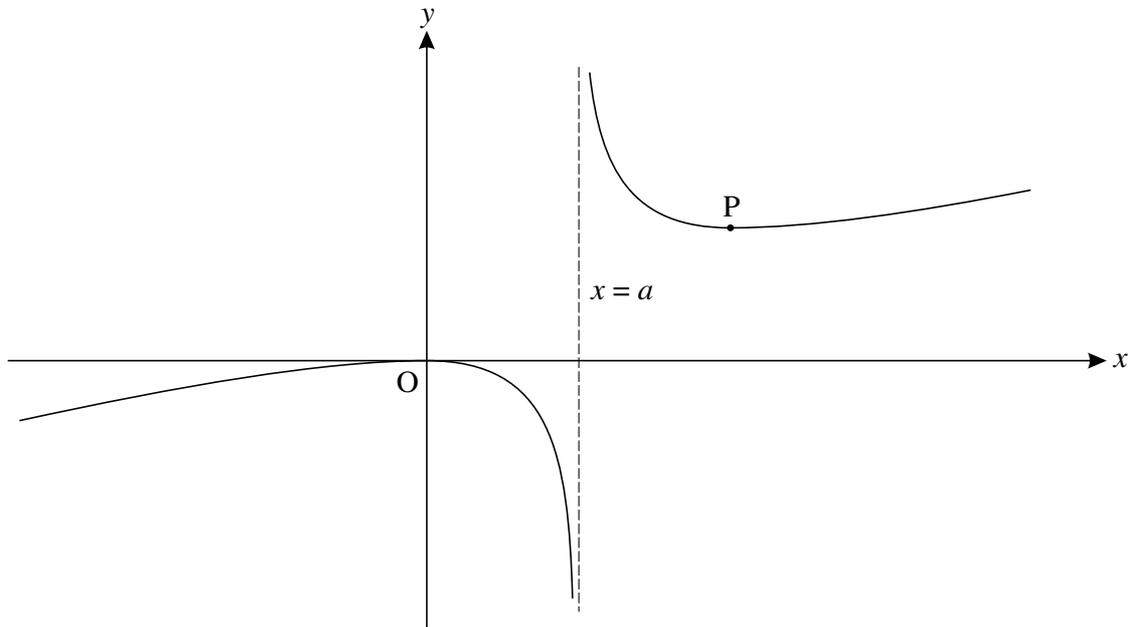


Fig. 9

- (i) Write down the value of a . [1]
- (ii) Show that $\frac{dy}{dx} = \frac{x(3x-2)}{(3x-1)^2}$. [3]
- (iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. [7]

- (iv) Using the substitution $u = 3x - 1$, show that $\int \frac{x^2}{3x-1} dx = \frac{1}{27} \int \left(u + 2 + \frac{1}{u} \right) du$.

Hence find the exact area of the region enclosed by the curve, the x -axis and the lines $x = \frac{2}{3}$ and $x = 1$. [7]

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