OCR
RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Show that $\int_1^2 \frac{1}{\sqrt{3x-2}}\,\mathrm{d}x = \frac{2}{3}$. **[5]**

2 Solve the inequality $|2x + 1| > 4$. **[3]**

3 Find the gradient at the point $(0, \ln 2)$ on the curve with equation $e^{2y} = 5 - e^{-x}$. **[4]**

4 Fig. 4 shows the curve $y = f(x)$, where $f(x) = \sqrt{1 - 9x^2}$, $-a \leqslant x \leqslant a$.

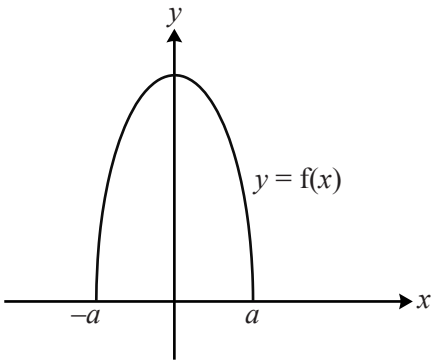

Fig. 4

(i) Find the value of a. **[2]**

(ii) Write down the range of $f(x)$. **[1]**

(iii) Sketch the curve $y = f(\frac{1}{3}x) - 1$. **[3]**

5 A termites' nest has a population of P million. P is modelled by the equation $P = 7 - 2e^{-kt}$, where t is in years, and k is a positive constant.

(i) Calculate the population when $t = 0$, and the long-term population, given by this model. **[3]**

(ii) Given that the population when $t = 1$ is estimated to be 5.5 million, calculate the value of k. **[3]**

6 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 2\arcsin x$, $-1 \leqslant x \leqslant 1$.

Fig. 6 also shows the curve $y = \mathrm{g}(x)$, where $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

P is the point on the curve $y = \mathrm{f}(x)$ with x-coordinate $\frac{1}{2}$.

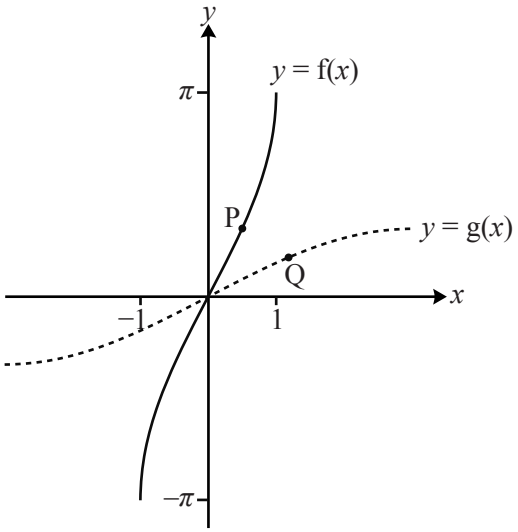

Fig. 6

(i) Find the y-coordinate of P, giving your answer in terms of π. **[2]**

The point Q is the reflection of P in $y = x$.

(ii) Find $\mathrm{g}(x)$ and its derivative $\mathrm{g}'(x)$. Hence determine the exact gradient of the curve $y = \mathrm{g}(x)$ at the point Q.

Write down the exact gradient of $y = \mathrm{f}(x)$ at the point P. **[6]**

7 You are given that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are odd functions, defined for $x \in \mathbb{R}$.

(i) Given that $\mathrm{s}(x) = \mathrm{f}(x) + \mathrm{g}(x)$, prove that $\mathrm{s}(x)$ is an odd function. **[2]**

(ii) Given that $\mathrm{p}(x) = \mathrm{f}(x)\mathrm{g}(x)$, determine whether $\mathrm{p}(x)$ is odd, even or neither. **[2]**

Section B (36 marks)

8 Fig. 8 shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x-axis at the point P. The tangent to the curve at P crosses the y-axis at Q.

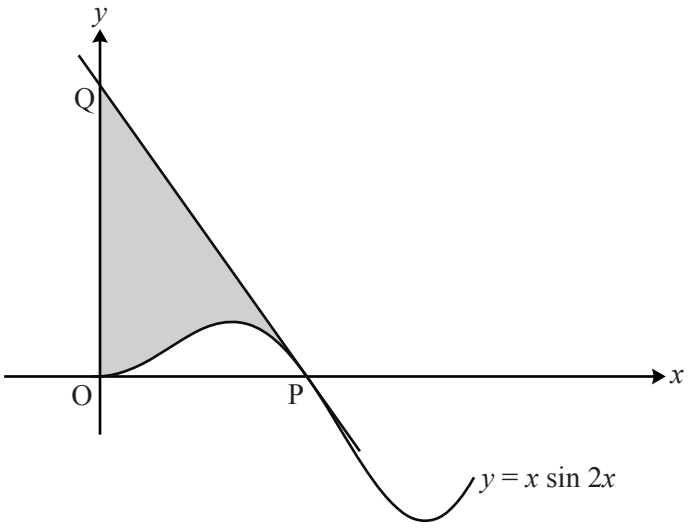

Fig. 8

(i) Find $\frac{dy}{dx}$. Hence show that the x-coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. **[4]**

(ii) Find, in terms of π, the x-coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. **[7]**

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. **[7]**

9 Fig. 9 shows the curve $y = f(x)$, which has a y-intercept at P(0, 3), a minimum point at Q(1, 2), and an asymptote $x = -1$.

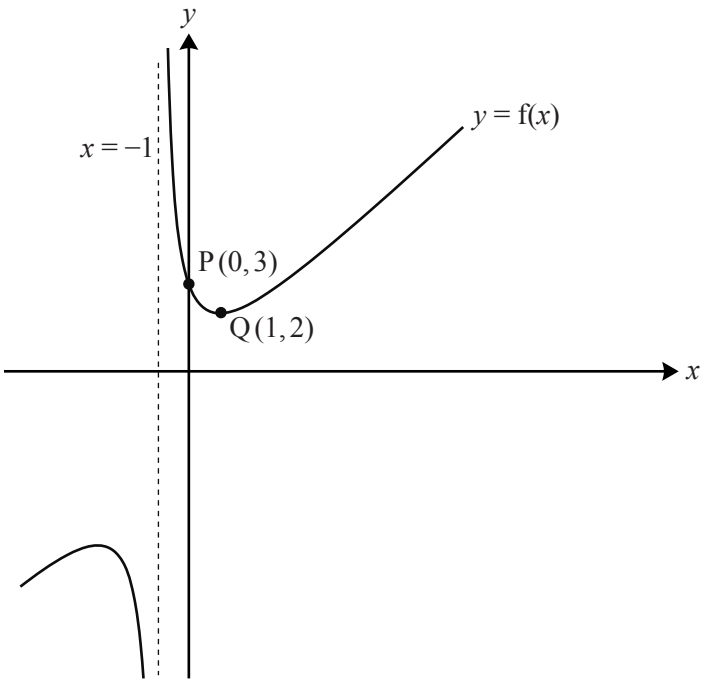

Fig. 9

(i) Find the coordinates of the images of the points P and Q when the curve $y = f(x)$ is transformed to

(A) $y = 2f(x)$,

(B) $y = f(x + 1) + 2$. **[4]**

You are now given that $f(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$.

(ii) Find $f'(x)$, and hence find the coordinates of the other turning point on the curve $y = f(x)$. **[6]**

(iii) Show that $f(x - 1) = x - 2 + \dfrac{4}{x}$. **[3]**

(iv) Find $\displaystyle\int_a^b \left(x - 2 + \frac{4}{x}\right) \mathrm{d}x$ in terms of a and b.

Hence, by choosing suitable values for a and b, find the exact area enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 1$. **[5]**

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