

# OCR

Oxford Cambridge and RSA

## Friday 12 June 2015 – Morning

### A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (36 marks)

- 1 Fig. 1 shows part of the curve  $y = e^{2x} \cos x$ .

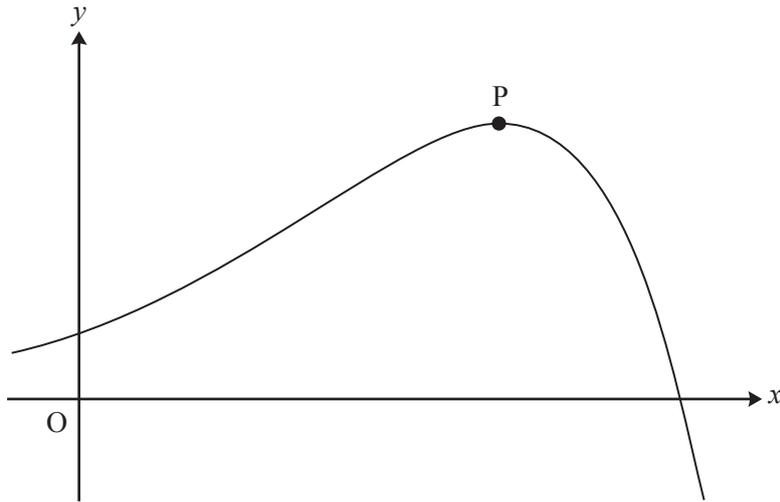


Fig. 1

Find the coordinates of the turning point P.

[6]

2 Find  $\int \sqrt[3]{2x-1} dx$ .

[4]

3 Find the exact value of  $\int_1^2 x^3 \ln x dx$ .

[5]

- 4 Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is  $45^\circ$ . Water is poured into the cone at a constant rate of  $5 \text{ cm}^3$  per second. At time  $t$  seconds, the height of the water surface above the vertex O of the cone is  $h$  cm, and the volume of water in the cone is  $V \text{ cm}^3$ .

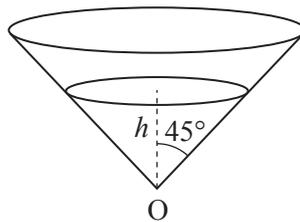


Fig. 4

Find  $V$  in terms of  $h$ .

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume  $V$  of a cone of height  $h$  and radius  $r$  is  $V = \frac{1}{3}\pi r^2 h$ ].

[5]

- 5 A curve has implicit equation  $y^2 + 2x \ln y = x^2$ .

Verify that the point  $(1, 1)$  lies on the curve, and find the gradient of the curve at this point.

[6]

6 Solve each of the following equations, giving your answers in exact form.

(i)  $6 \arcsin x - \pi = 0.$  [2]

(ii)  $\arcsin x = \arccos x.$  [2]

7 (i) The function  $f(x)$  is defined by

$$f(x) = \frac{1-x}{1+x}, x \neq -1.$$

Show that  $f(f(x)) = x.$

Hence write down  $f^{-1}(x).$  [3]

(ii) The function  $g(x)$  is defined for all real  $x$  by

$$g(x) = \frac{1-x^2}{1+x^2}.$$

Prove that  $g(x)$  is even. Interpret this result in terms of the graph of  $y = g(x).$  [3]

## Section B (36 marks)

- 8 Fig. 8 shows the line  $y = 1$  and the curve  $y = f(x)$ , where  $f(x) = \frac{(x-2)^2}{x}$ . The curve touches the  $x$ -axis at  $P(2, 0)$  and has another turning point at the point  $Q$ .

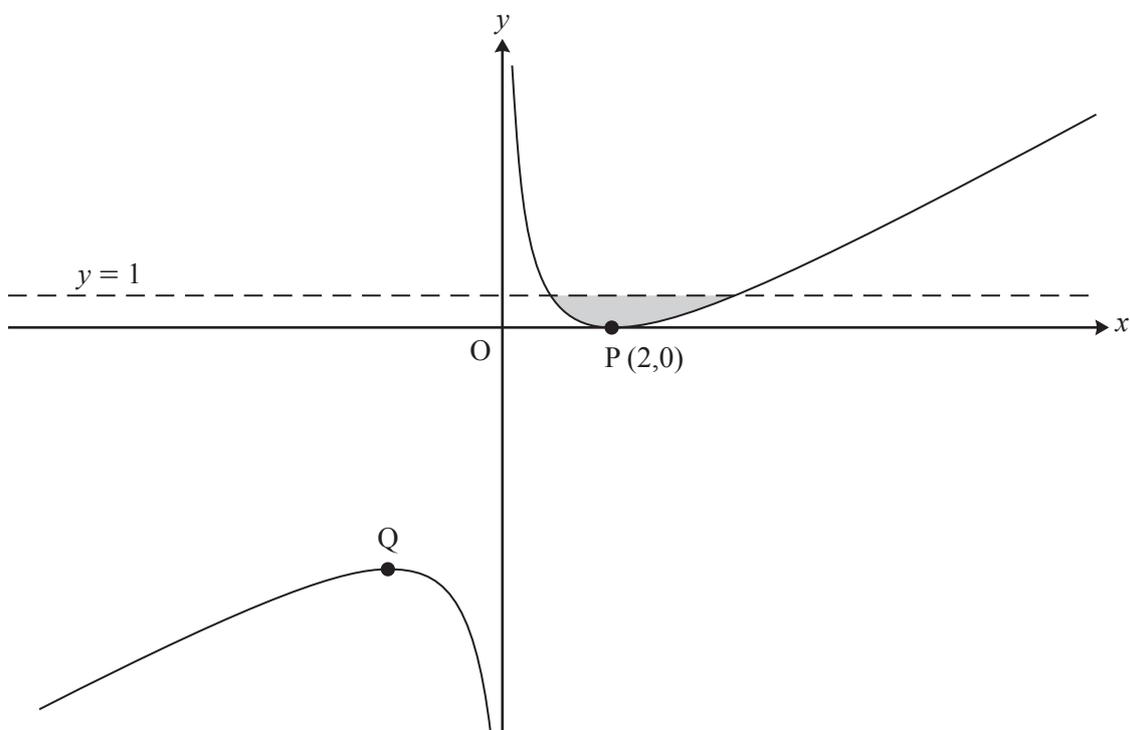


Fig. 8

- (i) Show that  $f'(x) = 1 - \frac{4}{x^2}$ , and find  $f''(x)$ .

Hence find the coordinates of  $Q$  and, using  $f''(x)$ , verify that it is a maximum point. [7]

- (ii) Verify that the line  $y = 1$  meets the curve  $y = f(x)$  at the points with  $x$ -coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. [6]

The curve  $y = f(x)$  is now transformed by a translation with vector  $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ . The resulting curve has equation  $y = g(x)$ .

- (iii) Show that  $g(x) = \frac{x^2 - 3x}{x + 1}$ . [3]

- (iv) Without further calculation, write down the value of  $\int_0^3 g(x) dx$ , justifying your answer. [2]

9 Fig. 9 shows the curve  $y = f(x)$ , where

$$f(x) = (e^x - 2)^2 - 1, \quad x \in \mathbb{R}.$$

The curve crosses the  $x$ -axis at O and P, and has a turning point at Q.

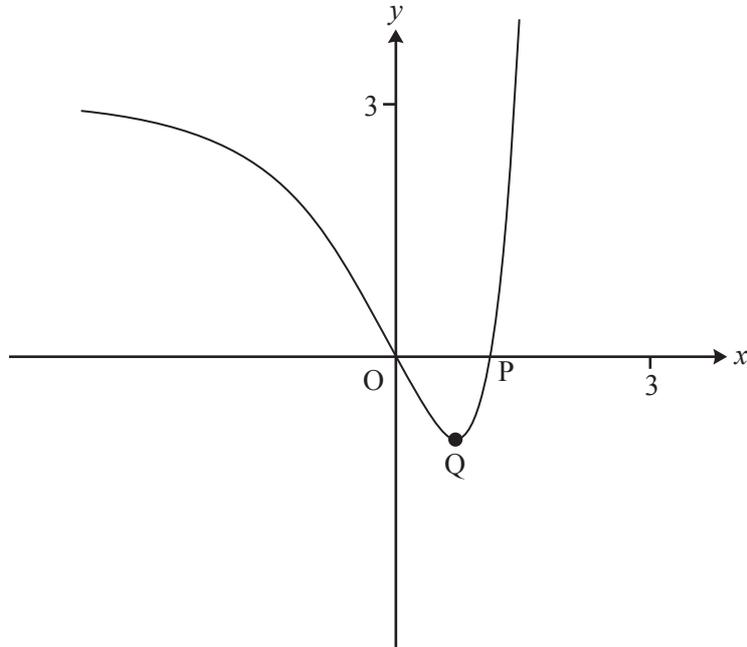


Fig. 9

- (i) Find the exact  $x$ -coordinate of P. [2]
- (ii) Show that the  $x$ -coordinate of Q is  $\ln 2$  and find its  $y$ -coordinate. [4]
- (iii) Find the exact area of the region enclosed by the curve and the  $x$ -axis. [5]

The domain of  $f(x)$  is now restricted to  $x \geq \ln 2$ .

- (iv) Find the inverse function  $f^{-1}(x)$ . Write down its domain and range, and sketch its graph on the copy of Fig. 9. [7]

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