OCR
Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $\dfrac{1}{(5-2x^3)^2}$. **[3]**

2 The function $f(x)$ is defined by $f(x)=|x|$, for $-1 \leqslant x \leqslant 1$.

Sketch the graph of $y=g(x)$, where $g(x)=2-2f(x)$. **[3]**

3 Functions $f(x)$ and $g(x)$, each defined for $-1 < x < 1$, are given by $f(x)=\ln(1-x)$ and $g(x)=x^2$.

(i) Find $f^{-1}(x)$ and state its domain and range. **[4]**

(ii) Show that $f(x)+f(-x)=fg(x)$. **[3]**

4 A curve has equation $3x^{\frac{2}{3}}+2y^{\frac{1}{3}}=7$.

(i) By differentiating implicitly, find $\dfrac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Hence find the gradient of the curve at the point with coordinates (1, 8). **[2]**

5 A liquid is being heated. At time t minutes after heating starts, its temperature, θ °C, is modelled by the equation

$$\theta = 10.5 + 69.5(1-e^{-kt}),$$

where k is a positive constant. The boiling point of the liquid is the value approached by θ as t tends to infinity.

(i) Write down the initial temperature and the boiling point of the liquid. **[2]**

(ii) After being heated for one minute, the liquid has a temperature of 30 °C. Find k. **[3]**

(iii) Find how long it takes from the start of the heating until the temperature is within 1 °C of the boiling point. Give your answer to the nearest minute. **[3]**

6 You are given that the sum of the interior angles of a polygon with n sides is $180(n-2)°$. Using this result, or otherwise, prove that the interior angle of a regular polygon cannot be 155°. **[3]**

7 The equation of a curve is $y=\arcsin\frac{1}{2}x$.

(i) Express each of x and $\dfrac{dx}{dy}$ in terms of y. **[2]**

(ii) A point is moving on the curve, and has coordinates (x, y) at time t. When $x=1$, the value of $\dfrac{dx}{dt}$ is 2.

Find the exact value of $\dfrac{dy}{dt}$ at this instant. **[5]**

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = \dfrac{\cos x}{2 - \sin x}$. The curve intersects the x- and y-axes at A and C respectively, and has a turning point at B.

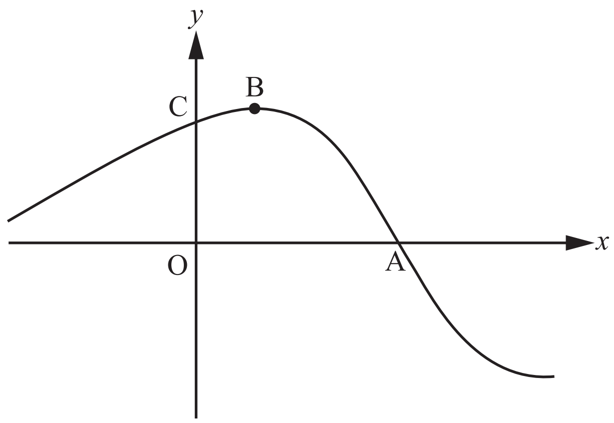

Fig. 8

(i) Write down the coordinates of A and C. **[2]**

(ii) Find $\dfrac{dy}{dx}$ and the exact coordinates of B. **[7]**

(iii) (*A*) Using integration by substitution, or otherwise, find the exact area of the region enclosed by the curve, the y-axis and the positive x-axis. **[4]**

(*B*) The line $x = k$ divides this region into two parts of equal area. Show that $k = \arcsin(2 - \sqrt{2})$. **[5]**

9 A curve has equation $y = f(x)$, where $f(x) = x^3 e^{-x^2}$.

(i) Show that $f(x)$ is an odd function, and interpret this result in terms of the graph of the curve $y = f(x)$. **[3]**

(ii) Find the coordinates of the stationary points of the curve. Give answers correct to 2 decimal places where appropriate. **[7]**

(iii) Sketch the curve for $-2 \leqslant x \leqslant 2$. **[2]**

(iv) (*A*) Show, using the substitution $t = x^2$, that $\int f(x)\,dx$ may be expressed as $\int kt\,e^{-t}\,dt$, where k is a constant to be determined. **[2]**

(*B*) Hence find the exact area of the region enclosed by the curve $y = f(x)$, the positive x-axis and the line $x = 2$. **[4]**

END OF QUESTION PAPER

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1

2	

3 (i)	
3 (ii)	

4 (i)	
4 (ii)	

5 (i)	
5 (ii)	
5 (iii)	
	(answer space continued on next page)

5 (iii)	**(continued)**
6	

7 (i)	
7 (ii)	

Section B (36 marks)

8 (i)	
8 (ii)	

8 (iii)(*A*)	
8 (iii)(*B*)	

9 (i)	
9 (ii)	
	(answer space continued on next page)

9 (ii)	**(continued)**
9 (iii)	

9 (iv)(*A*)	
9 (iv)(*B*)	

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

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GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

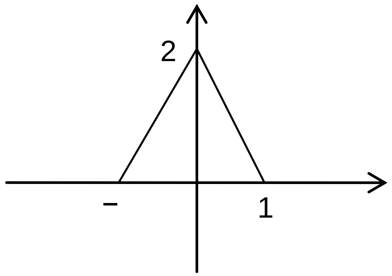
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

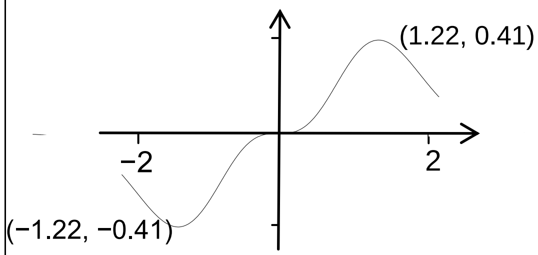
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\frac{1}{(5-2x^3)^2}=(5-2x^3)^{-2}$ $\frac{\mathrm{d}}{\mathrm{d}x}(5-2x^3)^{-2}=(-6x^2)(-2)(5-2x^3)^{-3}$ $=12x^2(5-2x^3)^{-3}$ isw	M1 A1 A1cao [3]	Chain rule on $(5-2x^3)^{-2}$ correct expression, allow $(-6x^2)(-2)u^{-3}$ o.e. or $\frac{12x^2}{(5-2x^3)^3}$ isw	or quotient (or product) e.g. $\frac{(5-2x^3)^2.0-1.2(5-2x^3)(-6x^2)}{(5-2x^3)^4}$ M1A1 [must have correct denom for M1] *u v*' – *v u*' in QR is M0
2			 	M1 A1 A1 [3]	inverted ‘v’ shape through (−1, 0), (1, 0) and (0, 2) correct domain ($-1 \le x \le 1$)	
3	(i)		$y = \ln(1-x)$ $x \leftrightarrow y$ $x = \ln(1-y)$ $\Rightarrow e^x = 1-y$ $\Rightarrow y = 1-e^x$ [so $f^{-1}(x) = 1-e^x$] domain $x < \ln 2$ range $-1 < y < 1$	M1 A1 B1 B1 [4]	or $e^y = 1-x$ $y = 1-e^x$ or $f^{-1}(x) = 1-e^x$ allow $x < 0.693$ or better, $-\infty < x < \ln 2$ or $-1 < f^{-1}(x) < 1$. Must use x for domain, y or $f^{-1}(x)$ for range.	can interchange x and y at any stage $x \le \ln 2$ is B0, $\ln 0 < x < \ln 2$ is B0 allow (–1, 1) but not [–1, 1]. If not labelled, take inequality with x as domain and with y or $f^{-1}(x)$ as range
3	(ii)		$f(-x) = \ln(1+x)$ $fg(x) = \ln(1-x^2)$ $\ln(1-x) + \ln(1+x) = \ln(1-x)(1+x)$ $= \ln(1-x^2)$	B1 B1 B1 [3]	soi e.g. from $\ln(1-x) + \ln(1+x) = \ldots$	must include brackets must include brackets

4	(i)		$2x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{2}{3}}\frac{dy}{dx}=0$ $\Rightarrow\frac{dy}{dx}=-3x^{-\frac{1}{3}}y^{\frac{2}{3}}$ o.e.	M1 A1 A1 [3]	$\frac{d}{dx}(y^{\frac{1}{3}})=\frac{1}{3}y^{-\frac{2}{3}}\frac{dy}{dx}$ seen correct equation must simplify 2 / (2/3) = 3	mark final answer
	(ii)		when $x = 1$, $y = 8$, $\Rightarrow\frac{dy}{dx}=-3\times4=-12$	M1 A1cao [2]	substituting both $x = 1$ and $y = 8$ into their dy/dx NB check power of x is correct in part (i)	
5	(i)		Initial temperature is 10.5 [°C] boiling point is 80[°C]	B1 B1 [2]		
5	(ii)		$30 = 10.5 + 69.5(1 – e^{-k})$, $\Rightarrow e^{-k} = 1 - 19.5/69.5$ $\Rightarrow -k = \ln(0.7194…)$ $\Rightarrow k = -\ln(0.7194…) = 0.3293…$	B1 M1 A1 [3]	re-arranging and taking lns (correctly) art 0.33 or ln(139/100) o.e.	
5	(iii)		$79 = 10.5 + 69.5(1 – e^{-kt})$ $\Rightarrow e^{-kt} = 1 – 0.9856..= 0.014388489...$ $\Rightarrow t = -\ln(0.014388…)/0.3293$ [= 12.879…] = 13 mins	M1 M1 A1cao [3]	substituting θ = their (80–1) into the eqn and rearranging for e^{-kt} taking lns correctly	Trial and error: e.g. $t = 12$, $\theta = 78.66$ $t = 13$, $\theta = 79.04$, so 13 mins SCB2
6			Suppose the polygon has n sides. Then $180(n – 2) = 155n$ $\Rightarrow 25n = 360$ [$\Rightarrow n = 14.4$] which is impossible as n is an integer So no regular polygon has interior angle 155° **or** When $n = 14$, int angle = 180×12/14 = 154.29° When $n = 15$, int angle = 180×13/15 = 156° So no n which gives an interior angle 155°.	M1 A1 A1cao B1 B1 B1 [3]	or sum of ext angles = 360° so $25n$ = 360 or 72/5 clear statement of conclusion accept 154°	

7	(i)		$\frac{1}{2}x = \sin y$ $\Rightarrow x = 2\sin y$ $\frac{dx}{dy} = 2\cos y$	B1cao B1cao [2]		
7	(ii)		$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$	M1	o.e.	
			When $x = 1$, $y = \arcsin\frac{1}{2} = \frac{\pi}{6}$ so $\frac{dx}{dy} = 2\cos\frac{\pi}{6} = \sqrt{3}$	M1 A1 A1	substituting $x = 1$ into $y = \arcsin \frac{1}{2}x$ $\pi/6$ or. $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$	condone 30° soi e.g. by $\frac{dy}{dt} = \frac{2}{\sqrt{3}}$
			or $\sin y = \frac{1}{2} \Rightarrow \cos y = \sqrt{1 - \sin^2 y}$ $= \sqrt{(1 - \frac{1}{4})} = \frac{\sqrt{3}}{2}$ $\Rightarrow \frac{dx}{dy} = \sqrt{3}$	M1 A1 A1	$\sqrt{(1 - \frac{1}{4})}$ soi, e.g. $\frac{dy}{dt} = \frac{2}{\sqrt{3}}$	
			or $\frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{1}{4}x^2}}.\frac{1}{2}$ when $x = 1$, $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$	M1A1 A1		
			$\Rightarrow 2 = \sqrt{3}.\frac{dy}{dt}$, $\frac{dy}{dt} = \frac{2}{\sqrt{3}}$	A1 [5]	or $\frac{2\sqrt{3}}{3}$	must be exact, but isw if approximated

8	(i)		$(\frac{\pi}{2}, 0)$, $(0, \frac{1}{2})$	B1B1 [2]	or $y = 0 \Rightarrow x = \pi/2$; $x = 0 \Rightarrow y = ½$ (isw)	Ignore incorrect labelling
8	(ii)		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2-\sin x)(-\sin x) - \cos x(-\cos x)}{(2-\sin x)^2}$ when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ (2–sin *x*)(−sin *x*)–cos *x*(−cos *x*)=0 $\Rightarrow 1 - 2\sin x = 0$ $\Rightarrow x = \frac{\pi}{6},\ y = \frac{\sqrt{3}}{3}$ o.e.	M1 A1 M1 M1 A1 A1 A1 [7]	correct quotient or product rule correct expression (isw) setting (only) their numerator to zero use of $\sin^2 x + \cos^2 x = 1$ must be exact, isw	denom must be correct at some stage missing brackets may be inferred from subsequent work not denominator withhold if denom is set to zero
8	(iii)	(*A*)	$\int_0^{\frac{\pi}{2}} \frac{\cos x}{2-\sin x}[\mathrm{d}x] = \left[-\ln(2-\sin x)\right]_0^{\frac{\pi}{2}}$	B1ft M1 A1	correct integral and limits *c* ln (2 – sin *x*) *c* = –1	ft their π/2 , not 90°, limits may be implied from subsequent work
			or let *u* = 2 – sin *x*, d*u*/d*x* = − cos *x* $= \int_2^1 -\frac{1}{u}\,\mathrm{d}u$ $= \left[-\ln u\right]_2^1$	 M1 A1	 $\int -\frac{1}{u}[\mathrm{d}u]$ (ignore limits) or $\int_1^2 \frac{1}{u}[\mathrm{d}u]$ [−ln *u*] (ignore limits) or $\left[\ln u\right]_1^2$	 or *u* = sin *x*, d*u*/d*x* = cos *x* $\int \frac{1}{2-u}[\mathrm{d}u]$ [–ln(2 – *u*)]
			= ln 2	A1 [4]	–ln (½) is A0, isw after ln 2	not ln 2 – ln 1
	(iii)	(*B*)	$\int_0^k \frac{\cos x}{2-\sin x}[\mathrm{d}x] = \frac{1}{2}\ln 2$ ⇒ ln 2 – ln(2 – sin *k*) = ½ ln 2 ln (2 – sin *k*) = ½ ln 2 = ln √2 ⇒ 2 – sin *k* = √2 ⇒ sin *k* = 2 – √2 ⇒ *k* = arcsin(2 – √2)*	M1 A1 M1 A1 A1cao	equating integral from 0 to *k* or from *k* to π/2 to ½ their area o.e. e.g. ln(2 – sin *k*) = ½ ln 2 eliminating logarithms correctly o.e. e.g $(2 - \sin k)^2 = 2$ **NB AG**	or equating integral from 0 to *k* to integral from *k* to π/2 ln 2 – ln(2 – sin *k*) = ln(2 – sin *k*) dep first M1
			or $\int_0^{\arcsin(2-\sqrt{2})} \frac{\cos x}{2-\sin x}[\mathrm{d}x] = \left[-\ln(2-\sin x)\right]_0^{\arcsin(2-\sqrt{2})}$ = ln 2 – ln(2 – 2 + √2)= ln 2 – ln√2 = ln 2 – ½ ln 2 = ½ ln 2*	 M1 A1 A1	SC: verifying (**max 3 marks out of 5**): attempt to find integral from 0 to arcsin(2–√2) correct expression **N.B AG**	 or from arcsin(2–√2) to π/2
				[5]		

9	(i)		$f(-x)=(-x)^3e^{-(-x)^2}$ $=-x^3e^{-x^2}=-f(x)$	M1 A1	substituting –*x* for *x* in f(*x*) must have $f(-x)=(-x)^3e^{-(-x)^2}$ for A1	at least once
			Rotational symmetry of order two about the origin.	B1 [3]	or point or half-turn (180°) symmetry about O	allow description of symmetry, e.g. 'fits its outline if rotated etc…'
9	(ii)		$f'(x)=3x^2e^{-x^2}+x^3(-2x)e^{-x^2}$	M1 A1*	product rule correct expression	consistent with their derivatives - condone deriv of e^{-x^2} is e^{-x^2} for M1
			f′(*x*) = 0 when $3x^2e^{-x^2}-2x^4e^{-x^2}=0$ $\Rightarrow 3x^2 = 2x^4$ $\Rightarrow x = 0, \sqrt{1.5}, -\sqrt{1.5}$ *y* = 0, 0.41. −0.41 So (0, 0), (1.22, 0.41), (−1.22, −0.41)	M1 M1 A1dep A2dep [7]	their deriv = 0 taking out or dividing by e^{-x^2} dep A1* or $x = \pm\sqrt{1.5}$ o.e. dep A1*	must be 2 terms must be 2 terms Allow SC A1 if both *x*-coords correct or one point correct (dep A1*)
9	(iii)			M1 A1dep [2]	correct shape for –2 ≤ *x* ≤ 2 with 2 TPs, through O, reasonable half turn symmetry coords of TPs and stationary inflexion at origin shown dep (0,0) given as a stationary point in part (ii)	need not show stationary inflexion at O. ignore shape outside –2 ≤ *x* ≤ 2 condone plotting beyond [–2, 2] provided shape is correct
9	(iv)	(*A*)	let $t = x^2$, d*t*/d*x* = 2*x* [⇒ *x*d*x* = ½ d*t*] o.e. $\int x^3e^{-x^2}[\mathrm{d}x]=\int x^2e^{-x^2}x[\mathrm{d}x]=\int \frac{1}{2}te^{-t}[\mathrm{d}t]$	M1 A1 [2]	 *k* = ½	
9	(iv)	(*B*)	$\int_0^2 x^3e^{-x^2}\,\mathrm{d}x = k\int_0^4 te^{-t}\,\mathrm{d}t$ let *u* = *t*, *v*′=e^{-t}, *u*′ = 1 , *v* = $-e^{-t}$ $=[k]\left\{\left[t(-e^{-t})\right]_0^4-\int_0^4(-e^{-t})\,\mathrm{d}t\right\}$ $=[k]\left\{\left[-e^{-t}-te^{-t}\right]_0^4\right\}$ $=-\frac{1}{2}e^{-4}-2e^{-4}+\frac{1}{2}=\frac{1}{2}-\frac{5}{2e^4}$	 M1 A1 A1 A1cao [4]	 correct parts on $\int te^{-t}[dt]$ or $\int kte^{-t}[dt]$ ignore limits, ft their *k* limits must be correct here, ft their *k* oe but must evaluate $e^0 = 1$ and combine e^{-4} terms	 ft their *k*, condone $v = e^{-t}$

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4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments:

The performance of candidates for this paper appears to have been broadly similar to recent years. Section A offered straightforward assessment of specification items, with the proof question found to be more accessible than in recent years, while the two section B questions contained more challenge.

The general standard of presentation of scripts was good, and there was little evidence that candidates had insufficient time to complete the examination. Pages 14-16 of the answer booklet were available for additional work, and used by a significant number of candidates, usually for multiple attempts at questions.

Comments on Individual Questions:

Section A

1 This was a straightforward test of the chain rule, in which over three quarters of the candidates scored full marks. Occasionally we saw a quotient rule used, which required to be simplified to gain full marks. Another occasional error was to get the wrong sign, e.g. $-12x^2(5-2x^3)^{-3}$.

2. Only a third of candidates scored all three marks here. The final mark required the domain of the graph to be correct – often the ‘v’ shape extended beyond $x = -1$ to 1. Other attempts bore no relation to the correct answer.

3(i) The first two marks for finding the inverse function were nearly always gained. However, accurate notation for the domain and the range was not often seen. Not many candidates scored full marks, with the domain proving particularly awkward to get right.

3(ii) Many candidates got all three marks here, though the structure of their ‘show’ was sometimes weak. Very occasionally we saw $\text{fg}(x) = \ln(1-x)^2$.

4(i) The implicit derivative here was a straightforward example, and virtually all the candidates got the derivative equation correct. However, simplifying the fractional expression to get the final mark was often missing or incorrect: in particular, many learners made mistakes when dividing 2 by 2/3.

4(ii) There were two easy marks here, and virtually all candidates achieved the ‘M’ mark for substituting for x and y in their derivative.

5(i) The initial temperature was almost always correct, but the boiling point was sometimes incorrect or missing, suggesting that the limit of e^{-kt} as t tends to infinity was not known.

5(ii) Exponential growth and decay equations are usually well answered, and this was no exception, with most candidates scoring full marks.

5(iii) This question depended on the boiling point being correct, so the facility was lower. However, over half the candidates got full marks. The solution was made considerably harder if an inequality was used, as the working needed to show the reversing of the inequality signs.

6 Candidates scored full marks or zero marks in roughly equal numbers here. Most gave the first method shown in the mark scheme, namely solving $180(n-2) = 155n$ to get $n = 14.4$, but we also

saw some examples of the second approach, finding the interior angles for 14 and 15 sides. By far the most common error was to solve $180(n – 2) = 155$, getting $n = 2.86$.

7(i) The majority of candidates scored both these marks. Occasionally they found dy/dx instead of dx/dy and lost a mark.

7(ii) Virtually all candidates wrote down a chain rule and scored 1 mark. Thereafter, many scored all the remaining marks. Errors were caused by muddling derivatives like dy/dx and dx/dy. Occasionally candidates attempted to use the derivative of arcsin x, though this was often incorrect through missing out the '½' factor.

Section B

8 Some candidates lost marks here from working in degrees rather than radians.

8(i) It is important that candidates state both the coordinates the right way round, so 'A = $\pi/2$ ' and 'B = ½ ' scored zero.

8(ii) Over half got full marks here. The quotient rule was well answered, and the subsequent simplification using $\sin^2 x + \cos^2 x = 1$ was good. The most common error was in the sign of the derivatives of sin x and cos x, which could fortuitously lead to the correct turning point – the final 'A' marks here being withheld in this case.

8(iii)(A) Most candidates used a substitution $u = 2 – \sin x$. Errors thereafter were $du/dx = \cos x$, or getting the limits the wrong way round, perhaps under the misconception that the larger number must be the upper limit of the integral).

8(iii)(B) This question was the most demanding in the paper, with nearly half the candidates scoring zero marks. Often the problem seemed to lie with getting expressions consistent with the limits of the integral, either in terms of x or u.

9(i) Most candidates scored 2 or 3 here. We required to see $f(–x) = (–x)^3 \exp(–x)^2$ in the proof that $f(x)$ was an odd function, with the brackets correctly placed. For the 'B' mark describing the property of the graph, we needed to see reference to 'symmetry', 'half-turn,180° or order 2', and 'about the origin'.

9(ii) The main problem with the product rule here was to get the correct derivative of $\exp(–x^2)$. A common mistake was to think this is $\exp(–x^2)$. Having found the correct derivative and equated it to zero, the next issue was dividing through by, or factorising, $\exp(–x^2)$. After this, not many candidates got all three turning points, either omitting the origin or (–1.22, –0.41) or both. Also, evaluating the y-coordinates was sometimes done incorrectly. Where these issues were overcome, half of the candidates scored 6 or over; of these, half scored full marks.

9(iii) Very few candidates scored both marks here. Many omitted the inflection at the origin, and the graphs were often lacking the point symmetry stated in part (i).

9(iv)(A) Half the candidates scored these two marks. Using a substitution in this context was perhaps unexpected.

9(iv)(B) They could get three out of the four marks with a missing, or incorrect, value for k, but not many succeeded with this.

Oxford Cambridge and RSA

Unit level raw mark and UMS grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0