

Mark Scheme 4754
January 2006

Section A

<p>1 $\frac{2x}{x-2} - \frac{4x}{x+1} = 3$</p> <p>$\Rightarrow 2x(x+1) - 4x(x-2) = 3(x-2)(x+1)$</p> <p>$\Rightarrow 2x^2 + 2x - 4x^2 + 8x = 3x^2 - 3x - 6$</p> <p>$\Rightarrow 0 = 5x^2 - 13x - 6$</p> <p>$\quad = (5x+2)(x-3)$</p> <p>$\Rightarrow x = -2/5 \text{ or } 3.$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>cao</p> <p>[5]</p>	<p>Clearing fractions expanding brackets oe</p> <p>factorising or formula</p>
<p>2 $\frac{dx}{dt} = 1 - 1/t$</p> <p>$\frac{dy}{dt} = 1 + 1/t$</p> <p>$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$</p> <p>$\quad = \frac{1 + \frac{1}{t}}{1 - \frac{1}{t}}$</p> <p>When $t = 2$, $\frac{dy}{dx} = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = 3$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>Either dx/dt or dy/dt soi</p> <p>www</p>
<p>3 $\overline{BA} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}, \overline{BC} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$</p> <p>$\overline{BA} \cdot \overline{BC} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = (-4) \times 2 + 1 \times 5 + (-3) \times (-1)$</p> <p>$\quad = -8 + 5 + 3 = 0$</p> <p>$\Rightarrow \text{angle } ABC = 90^\circ$</p> <p>Area of triangle = $\frac{1}{2} \times BA \times BC$</p> <p>$\quad = \frac{1}{2} \times \sqrt{(-4)^2 + 1^2 + 3^2} \times \sqrt{2^2 + 5^2 + (-1)^2}$</p> <p>$\quad = \frac{1}{2} \times \sqrt{26} \times \sqrt{30}$</p> <p>$\quad = 13.96 \text{ sq units}$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>soi , condone wrong sense</p> <p>scalar product</p> <p>= 0</p> <p>area of triangle formula oe</p> <p>length formula</p> <p>accept 14.0 and $\sqrt{195}$</p>

<p>4(i) $2\sin 2\theta + \cos 2\theta = 1$ $\Rightarrow 4\sin \theta \cos \theta + 1 - 2\sin^2 \theta = 1$ $\Rightarrow 2\sin \theta (2\cos \theta - \sin \theta) = 0$ or $4 \tan \theta - 2 \tan^2 \theta = 0$</p> <p>$\Rightarrow \sin \theta = 0$ or $\tan \theta = 0, \theta = 0^\circ, 180^\circ$ or $2\cos \theta - \sin \theta = 0$ $\Rightarrow \tan \theta = 2$ $\Rightarrow \theta = 63.43^\circ, 243.43^\circ$</p> <p>OR Using $R\sin(2\theta + \alpha)$ $R = \sqrt{5}$ and $\alpha = 26.57^\circ$ $2\theta + 26.57 = \arcsin 1/R$ $\theta = 0^\circ, 180^\circ$ $\theta = 63.43^\circ, 243.43^\circ$</p>	<p>M1 A1 A1</p> <p>M1 A1, A1 [6]</p> <p>M1 A1 M1 A1 A1,A1 [6]</p>	<p>Using double angle formulae Correct simplification to factorisable or other form that leads to solutions 0° and 180°</p> <p>$\tan \theta = 2$ (-1 for extra solutions in range)</p> <p>(-1 for extra solutions in range)</p>
<p>5 (i) Plane has equation $x - y + 2z = c$ At $(2, -1, 4), 2 + 1 + 8 = c$ $\Rightarrow c = 11.$</p> <p>(ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 + \lambda \\ 12 + 3\lambda \\ 9 + 2\lambda \end{pmatrix}$</p> <p>$\Rightarrow 7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$ $\Rightarrow 2\lambda = -2$ $\Rightarrow \lambda = -1$ Coordinates are $(6, 9, 7)$</p>	<p>B1 M1 A1</p> <p>M1</p> <p>M1</p> <p>A1 A1 [7]</p>	<p>$x - y + 2z = c$ finding c</p> <p>ft their equation from (i)</p> <p>ft their $x - y + 2z = c$ cao</p>
<p>6 (i) $\frac{1}{\sqrt{4-x^2}} = 4^{-\frac{1}{2}}(1 - \frac{1}{4}x^2)^{-\frac{1}{2}}$</p> <p>$= \frac{1}{2} [1 + (-\frac{1}{2})(-\frac{1}{4}x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{1}{4}x^2)^2 + \dots]$</p> <p>$= \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \dots$</p> <p>(ii) $\int_0^1 \frac{1}{\sqrt{4-x^2}} dx \approx \int_0^1 (\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4) dx$</p> <p>$= \left[\frac{1}{2}x + \frac{1}{48}x^3 + \frac{3}{1280}x^5 \right]_0^1$</p> <p>$= \frac{1}{2} + \frac{1}{48} + \frac{3}{1280}$ $= 0.5232$ (to 4 s.f.)</p> <p>(iii) $\int_0^1 \frac{1}{\sqrt{4-x^2}} dx = \left[\arcsin \frac{x}{2} \right]_0^1$ $= \pi/6 = 0.5236$</p>	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p>M1ft</p> <p>A1</p> <p>B1 [7]</p>	<p>Binomial coeffs correct Complete correct expression inside bracket</p> <p>cao</p>

Section B

<p>7(i) $\hat{AOP} = 180 - \beta = 180 - \alpha - \theta$ $\Rightarrow \beta = \alpha + \theta$ $\Rightarrow \theta = \beta - \alpha$</p> $\tan \theta = \frac{\tan(\beta - \alpha)}{1 + \tan \beta \tan \alpha}$ $= \frac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10} \cdot \frac{y}{16}}$ $= \frac{16y - 10y}{160 + y^2}$ $= \frac{6y}{160 + y^2} *$ <p>When $y = 6$, $\tan \theta = 36/196$ $\Rightarrow \theta = 10.4^\circ$</p>	<p>M1 M1 E1</p> <p>M1 A1</p> <p>E1</p> <p>M1 A1 cao [8]</p>	<p>Use of sum of angles in triangle OPT and AOP oe</p> <p>SC B1 for $\beta = \alpha + \theta$, $\theta = \beta - \alpha$ no justification</p> <p>Use of Compound angle formula</p> <p>Substituting values for $\tan \alpha$ and $\tan \beta$</p> <p>www</p> <p>accept radians</p>
<p>(ii) $\sec^2 \theta \frac{d\theta}{dy} = \frac{(160 + y^2)6 - 6y \cdot 2y}{(160 + y^2)^2}$</p> $= \frac{6(160 + y^2 - 2y^2)}{(160 + y^2)^2}$ <p>$\Rightarrow \frac{d\theta}{dy} = \frac{6(160 - y^2)}{(160 + y^2)^2} \cos^2 \theta *$</p>	<p>M1</p> <p>M1 A1 A1</p> <p>E1</p> <p>[5]</p>	<p>$\sec^2 \theta \frac{d\theta}{dy} = \dots$</p> <p>quotient rule correct expression simplifying numerator www</p>
<p>(iii) $d\theta/dy = 0$ when $160 - y^2 = 0$ $\Rightarrow y^2 = 160$ $\Rightarrow y = 12.65$</p> <p>When $y = 12.65$, $\tan \theta = 0.237\dots$ $\Rightarrow \theta = 13.3^\circ$</p>	<p>M1</p> <p>A1</p> <p>M1 A1 cao [4]</p>	<p>oe</p> <p>accept radians</p>

<p>8 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow dx/dt = -ka(1 + kt)^{-2}$</p> $= -ka(x/a)^2$ $= -kx^2/a *$ <p>OR $kt = a/x - 1, t = a/kx - 1/k$ $dt/dx = -a/kx^2$</p> $\Rightarrow dx/dt = -kx^2/a$	<p>M1 A1</p> <p>E1 [3] M1 A1</p> <p>E1 [3]</p>	<p>Chain rule (or quotient rule)</p> <p>Substitution for x</p>
<p>(ii) When $t = 0, x = a \Rightarrow a = 2.5$ When $t = 1, x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$</p> $\Rightarrow 1 + k = 1.5625$ $\Rightarrow k = 0.5625$	<p>B1 M1</p> <p>A1 [3]</p>	<p>$a = 2.5$</p>
<p>(iii) In the long term, $x \rightarrow 0$</p>	<p>B1 [1]</p>	<p>or, for example, they die out.</p>
<p>(iv) $\frac{1}{2y - y^2} = \frac{1}{y(1 - y)} = \frac{A}{y} + \frac{B}{2 - y}$</p> $\Rightarrow 1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = 1/2$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = 1/2$ $\Rightarrow \frac{1}{2y - y^2} = \frac{1}{2y} + \frac{1}{2(2 - y)}$	<p>M1</p> <p>M1 A1 A1</p> <p>[4]</p>	<p>partial fractions</p> <p>evaluating constants by substituting values, equating coefficients or cover-up</p>
<p>(v) $\int \frac{1}{2y - y^2} dy = \int dt$</p> $\Rightarrow \int \left[\frac{1}{2y} + \frac{1}{2(2 - y)} \right] dy = \int dt$ $\Rightarrow \frac{1}{2} \ln y - \frac{1}{2} \ln(2 - y) = t + c$ When $t = 0, y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow \ln y - \ln(2 - y) = 2t$ $\Rightarrow \ln \frac{y}{2 - y} = 2t *$ $\frac{y}{2 - y} = e^{2t}$ $\Rightarrow y = 2e^{2t} - ye^{2t}$ $\Rightarrow y + ye^{2t} = 2e^{2t}$ $\Rightarrow y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow y = \frac{2e^{2t}}{1 + e^{2t}} = \frac{2}{1 + e^{-2t}} *$	<p>M1</p> <p>B1 ft</p> <p>A1</p> <p>E1</p> <p>M1</p> <p>DM1</p> <p>E1 [7]</p>	<p>Separating variables</p> <p>$\frac{1}{2} \ln y - \frac{1}{2} \ln(2 - y)$ ft their A,B</p> <p>evaluating the constant</p> <p>Anti-logging</p> <p>Isolating y</p>
<p>(vi) As $t \rightarrow \infty e^{-2t} \rightarrow 0 \Rightarrow y \rightarrow 2$ So long term population is 2000</p>	<p>B1 [1]</p>	<p>or $y = 2$</p>

Comprehension

1. It is the largest number in the Residual column in Table 5. **B1**

2. (i)

Acceptance percentage, $a\%$		10%	14%	12%	11%	10.5%
Party	Votes (%)	Seats	Seats	Seats	Seats	Seats
P	30.2	3	2	2	2	2
Q	11.4	1	0	0	1	1
R	22.4	2	1	1	2	2
S	14.8	1	1	1	1	1
T	10.9	1	0	0	0	1
U	10.3	1	0	0	0	0
Total seats		9	4	4	6	7

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

10% & 14% **B1**
Trial

M1

10.5% (10.3 < x ≤ 10.9) **A1**
Allocation **A1**

(ii)

Party	Round							Residual
	1	2	3	4	5	6	7	
P	30.2	15.1	15.1	10.07	10.07	10.07	10.07	10.07
Q	11.4	11.4	11.4	11.4	11.4	5.7	5.7	5.7
R	22.4	22.4	11.2	11.2	11.2	11.2	7.47	7.47
S	14.8	14.8	14.8	14.8	7.4	7.4	7.4	7.4
T	10.9	10.9	10.9	10.9	10.9	10.9	10.9	5.45
U	10.3	10.3	10.3	10.3	10.3	10.3	10.3	10.3
Seat allocated to	P	R	P	S	Q	R	T	

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

General method **M1** Round 2 correct **A1** Round 5 correct **A1**(condone minor arithmetic error) Residuals **A1** www Allocation **A1** cso

3.
$$\frac{11.2}{1+1} < 11 \leq \frac{11.2}{1} \Rightarrow 5.6 < 11 \leq 11.2$$

M1, A1

for either or both
M1 only for $5.6 < a \leq 11.2$

4. (i) The end-points of the intervals are the largest values in successive columns of Table 5.(or two largest within a column)

B1

So in

2	$16.6 < a \leq 22.2$
---	----------------------

22.2 is the largest number in Round 2. 16.6 is the largest number in Round 3.

B1

- (ii)

Seats	a	Seats	a
1	$22.2 < a \leq 27.0$	5	$11.1 < a \leq 11.2$
2	$16.6 < a \leq 22.2$	6	$10.6 < a \leq 11.1$
3	$13.5 < a \leq 16.6$	7	$9.0 < a \leq 10.6$
4	$11.2 < a \leq 13.5$		

5. (i) ● means \leq , ○ means $<$ (greater or less than)

B1

(ii)
$$\frac{V_k}{N_k + 1} < a \qquad a \leq \frac{V_k}{N_k}$$

$$V_k < aN_k + a \qquad aN_k \leq V_k$$

$$V_k - aN_k < a \qquad 0 \leq V_k - aN_k$$

$$0 \leq V_k - aN_k < a$$

B1

- (iii) The unused votes may be zero but must be less than a .

B1